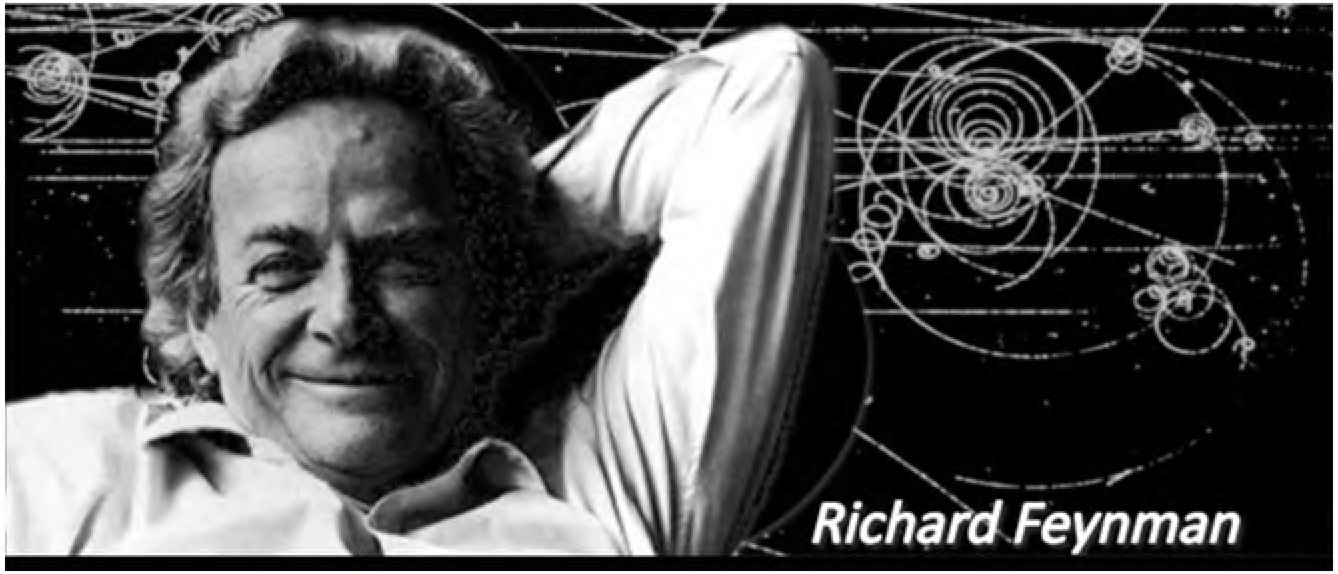
Fundamentals of Physics II

Faculty of Physics-Kharazmi University

Dr. Faramarz Kanjouri

Spring 2023



اگر همواره مانند گذشته بیندیشید، همیشه همان چیزهایی را به‌دست می‌آورید که تا کنون کسب کرده‌اید

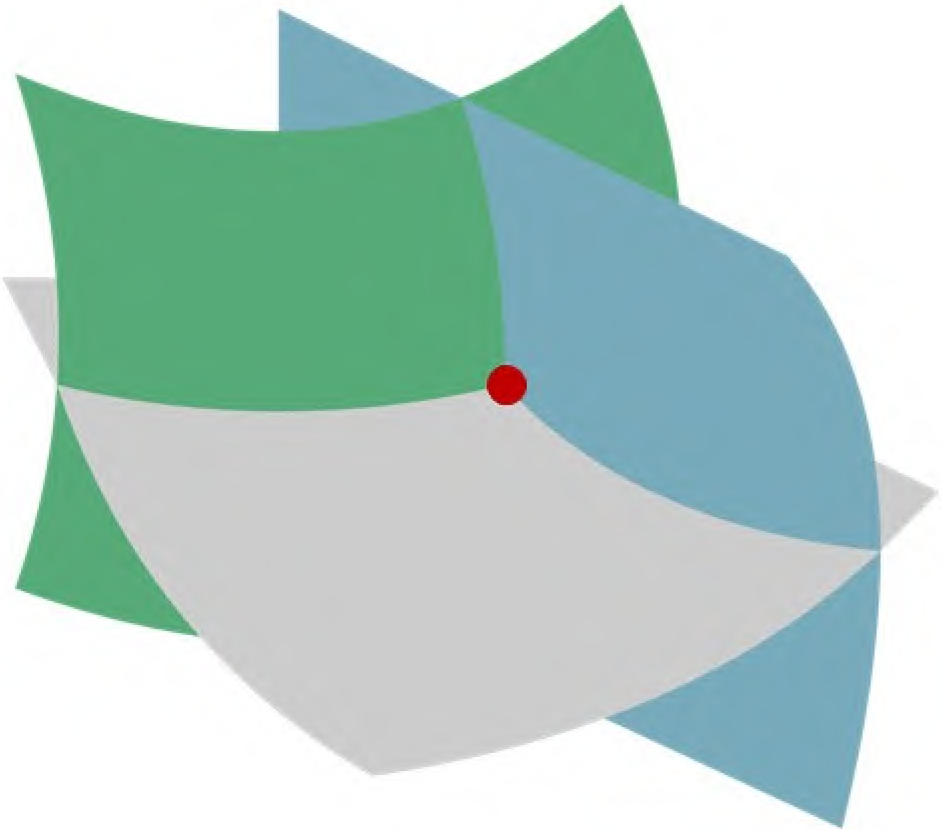
فاینمن

درس چهارم

دستگاه‌های مختصات- بخش۱

دانشگاه خوارزمی

- دستگاه مختصات چیست؟
- معرفی کلی دستگاه‌های مختصات کارتزین، کروی و استوانه‌ای
- عنصر طول و سطح در دستگاه کارتزین دو بعدی
- عنصر طول و سطح در دستگاه قطبی دو بعدی
- رابطه‌ی بین دستگاه‌های مختصات قطبی و کارتزین (دو بعدی)



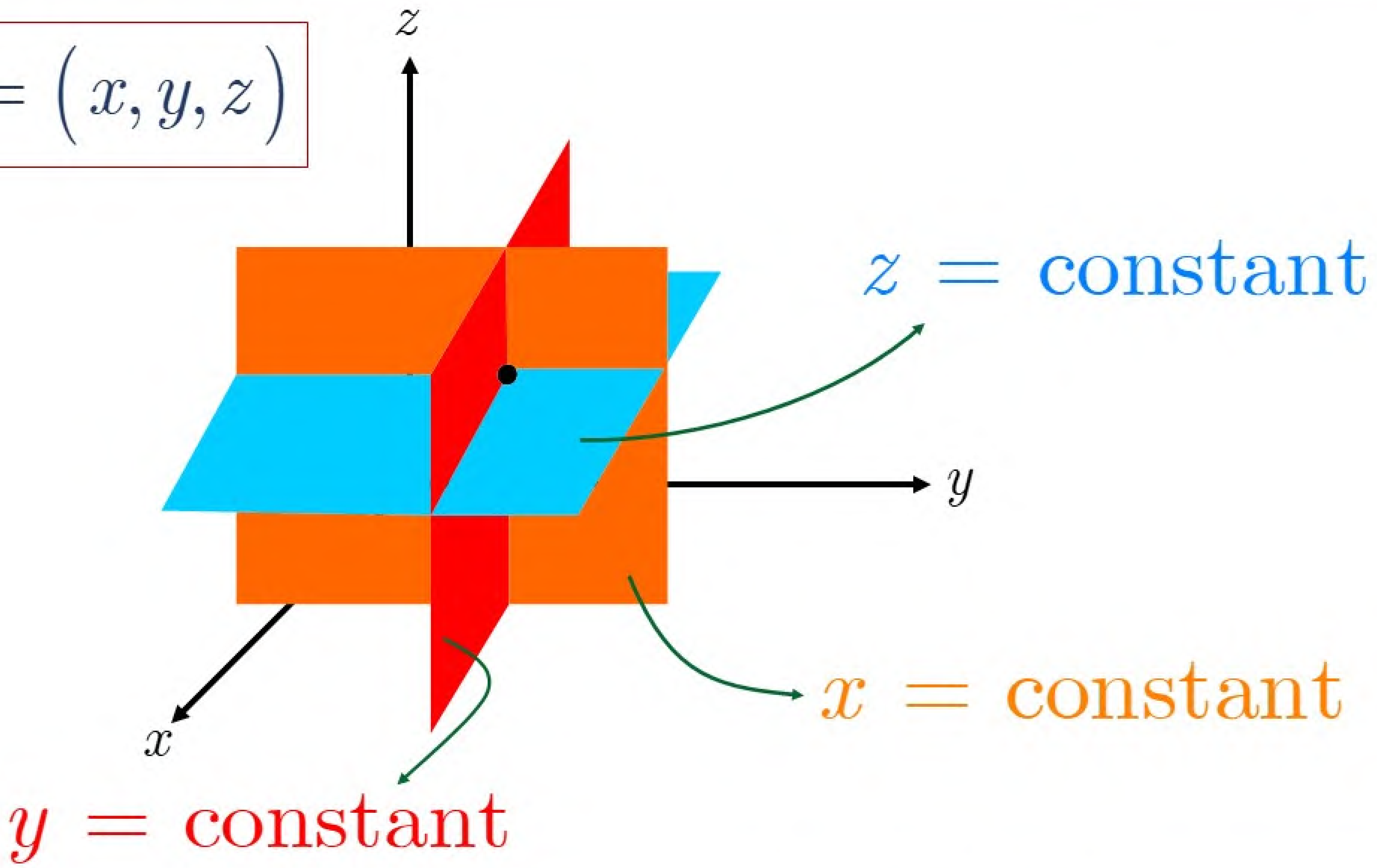
$u_1 = \text{constant}$

$u_2 = \text{constant}$

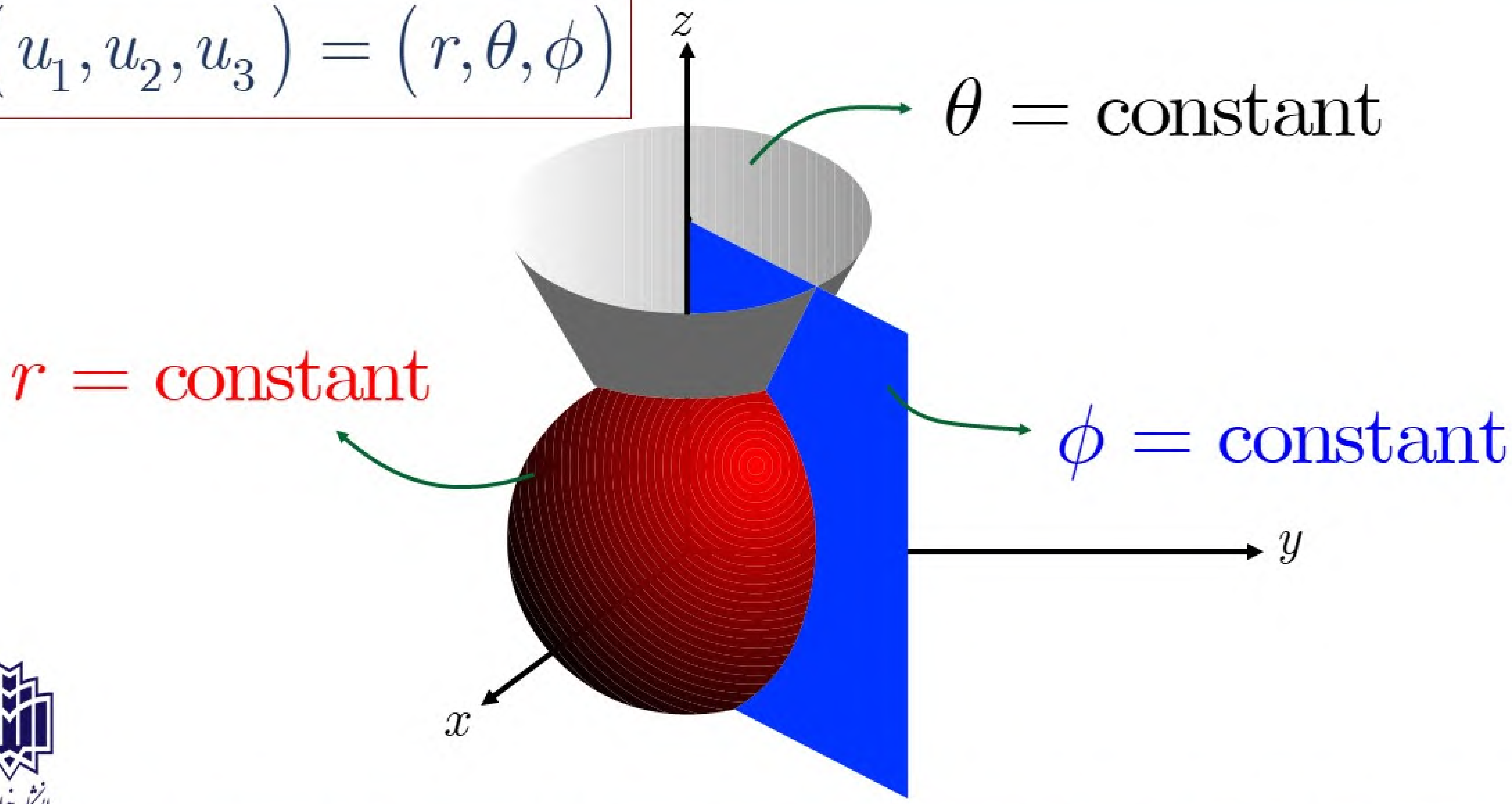
$u_3 = \text{constant}$

دانشگاه خوارزمی

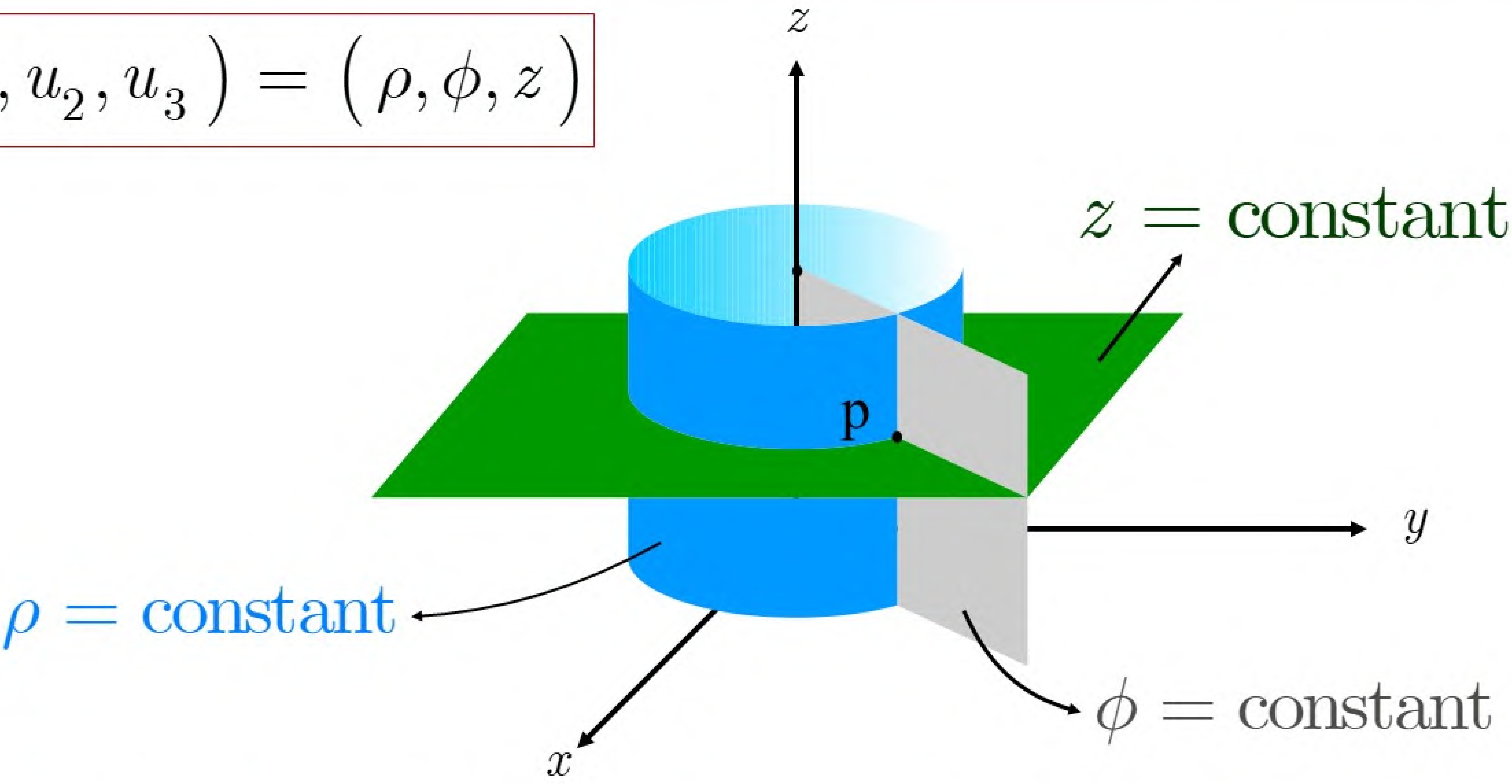
$$\left(u_1, u_2, u_3\right) = \left(x, y, z\right)$$

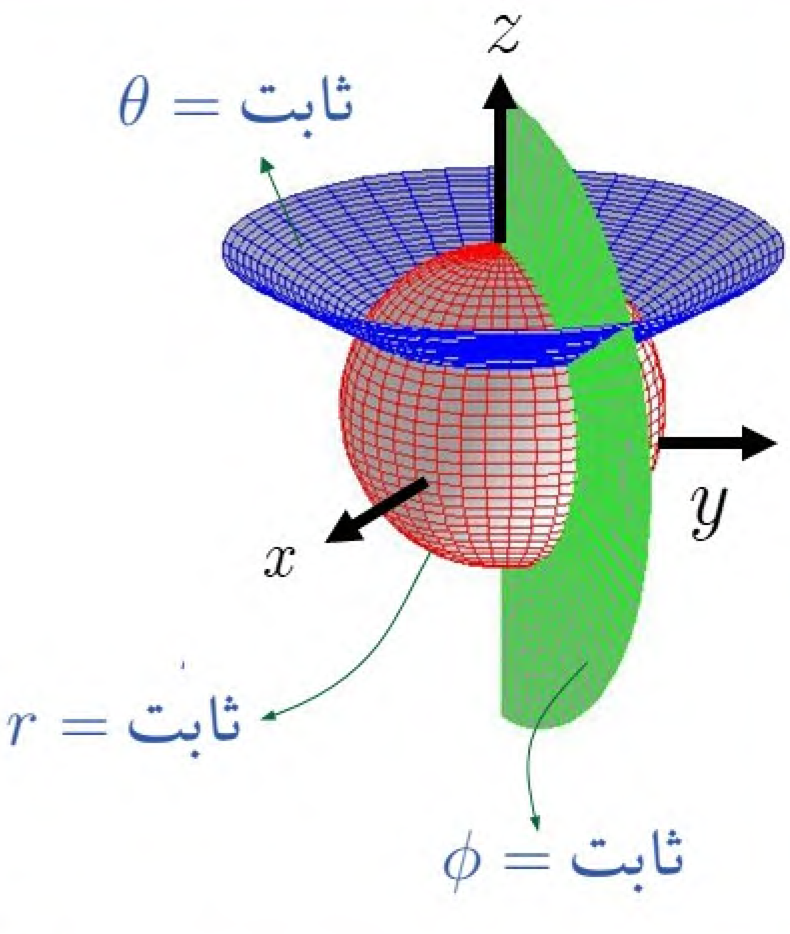


$$\left(u_1, u_2, u_3\right) = \left(r, \theta, \phi\right)$$

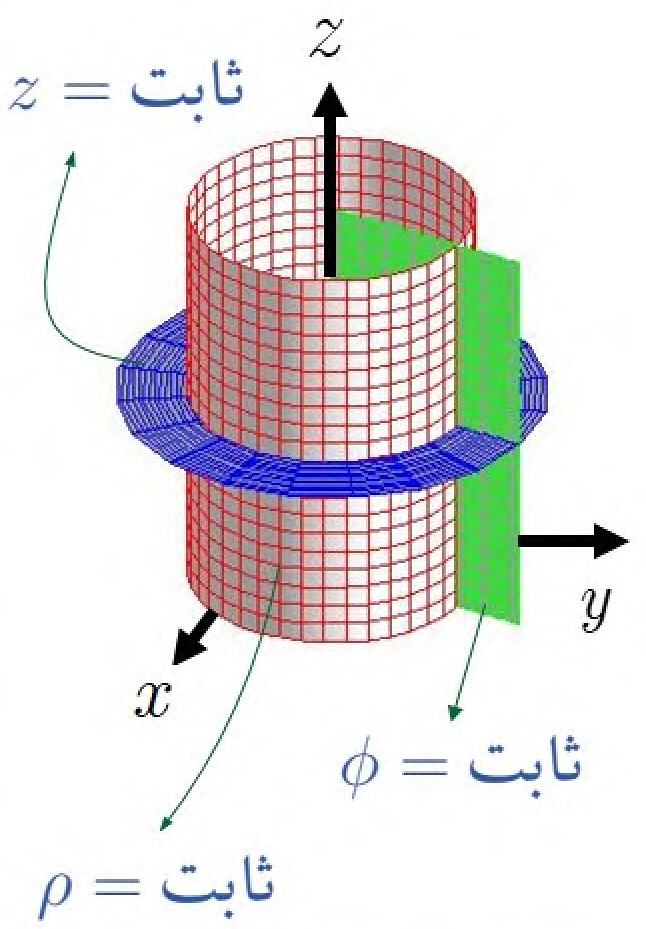


$$\left(u_1, u_2, u_3\right) = \left(\rho, \phi, z\right)$$

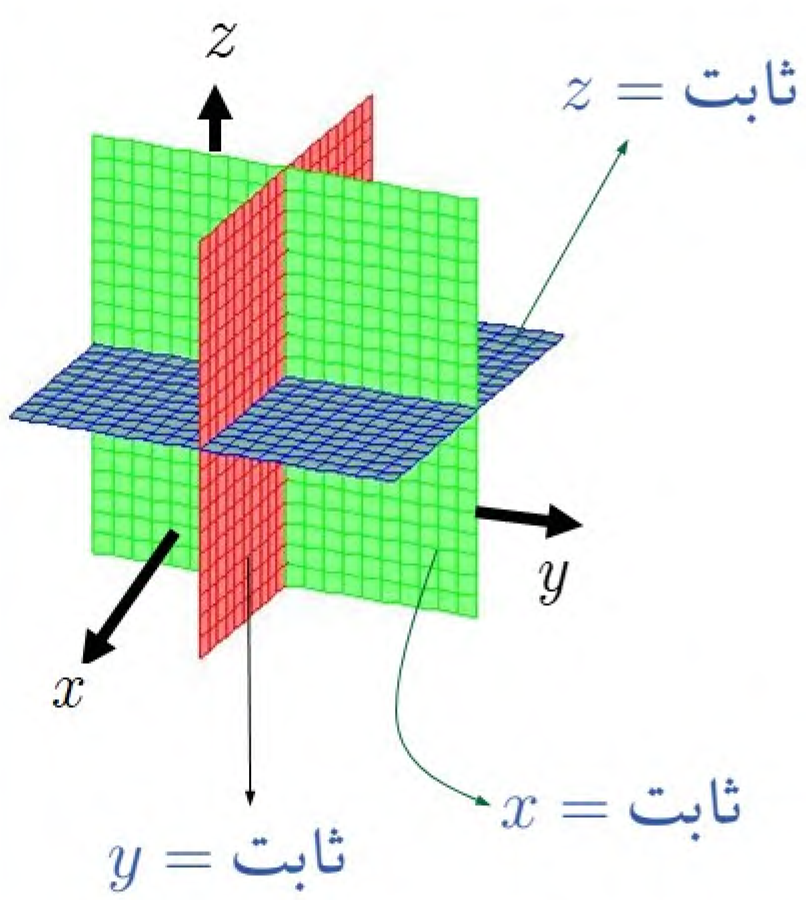




$$\left(u_1, u_2, u_3\right) = \left(r, \theta, \phi\right)$$

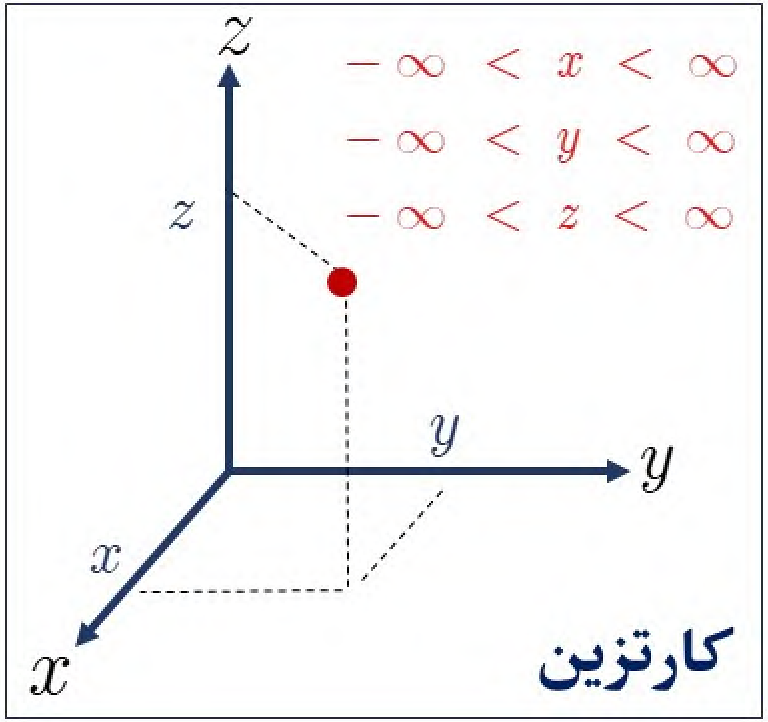


$$\left(u_1, u_2, u_3\right) = \left(\rho, \phi, z\right)$$

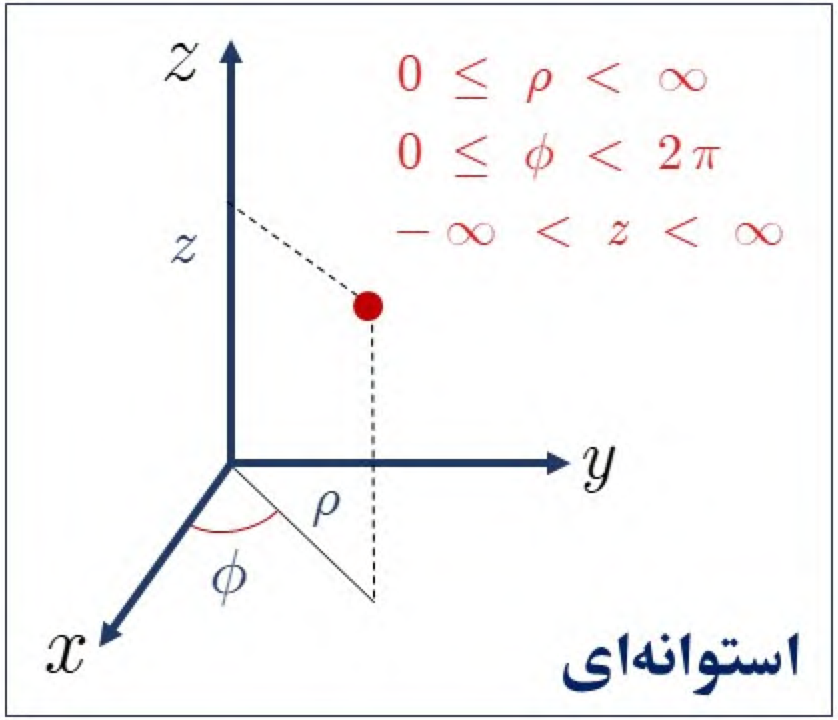


$$\left(u_1, u_2, u_3\right) = \left(x, y, z\right)$$

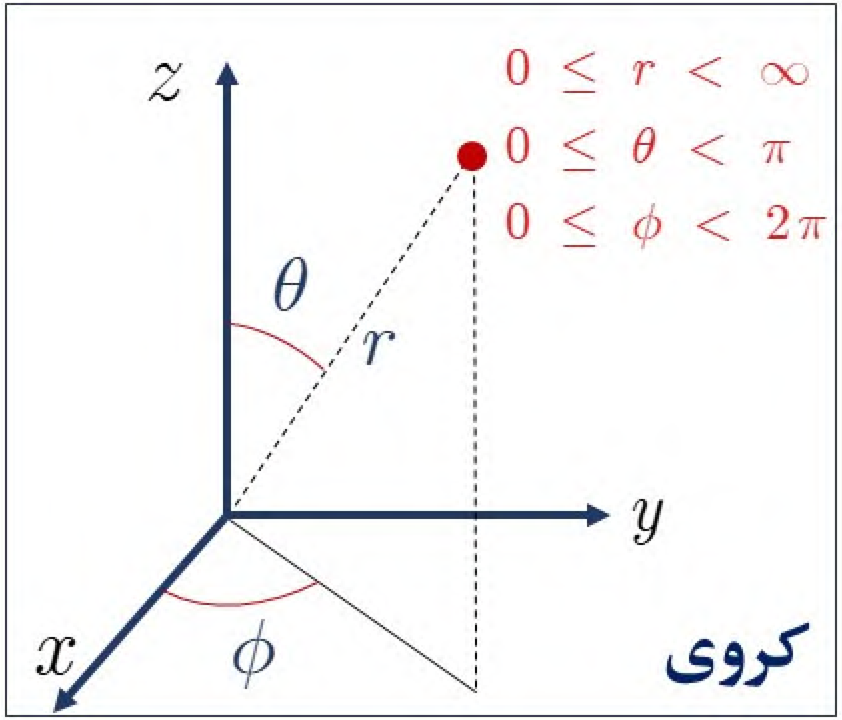
دانشگاه خوارزمی



$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$
z
x
y
کارتزین

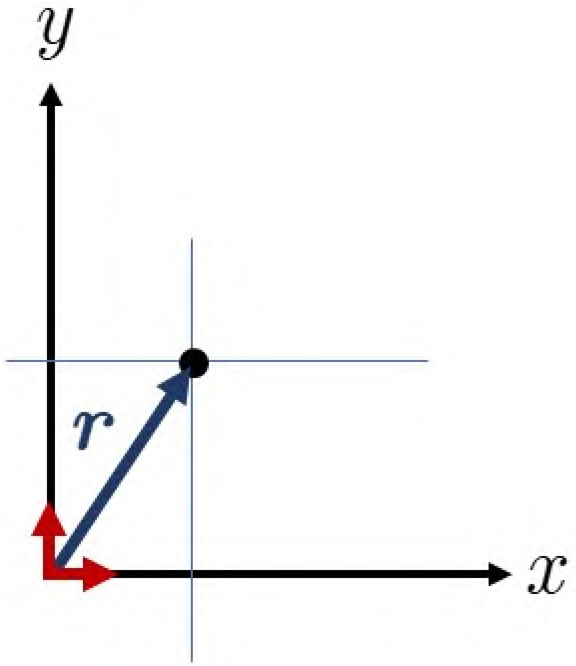


$0 \leq \rho < \infty$
$0 \leq \phi < 2\pi$
$-\infty < z < \infty$
z
x
y
ρ
ϕ
استوانه‌ای

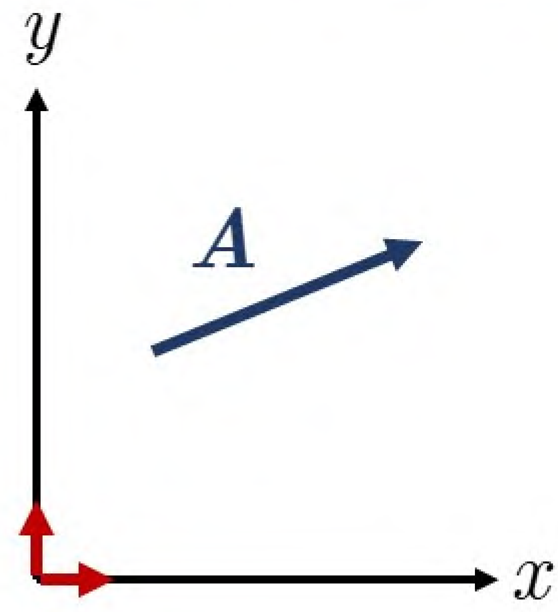


$0 \leq r < \infty$
$0 \leq \theta < \pi$
$0 \leq \phi < 2\pi$
z
x
y
θ
r
ϕ
کروی

برای درک بهتر مسئله، حالت دو بعدی را بررسی می کنیم



$$\boldsymbol{r} = x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y$$

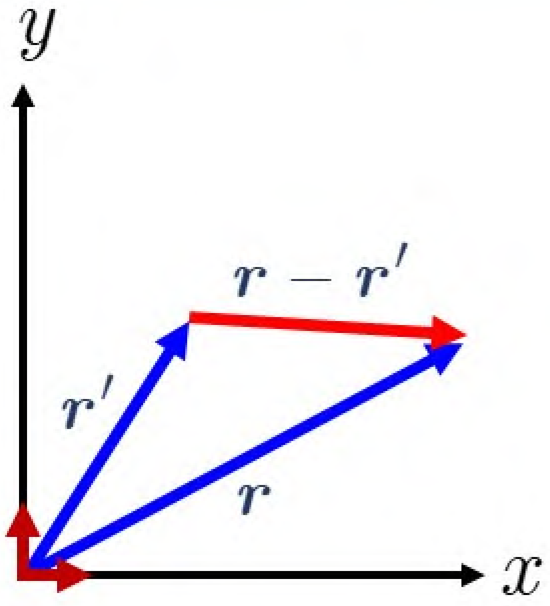


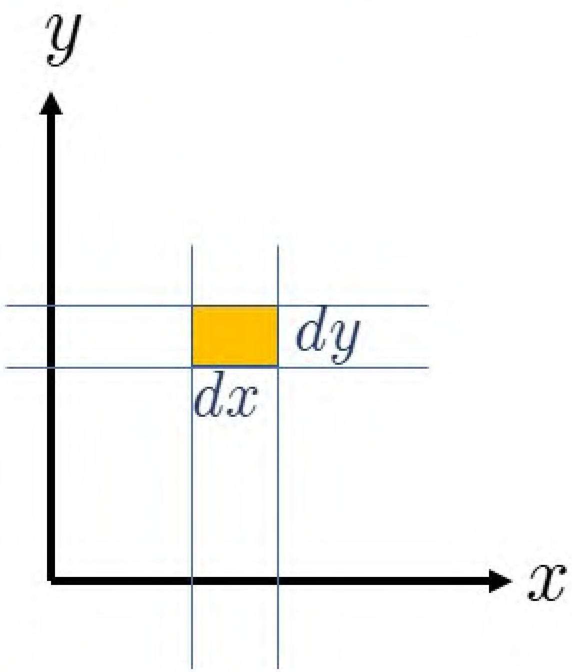
$$\boldsymbol{A} = A_x\hat{\boldsymbol{e}}_x + A_y\hat{\boldsymbol{e}}_y$$

$$A_x = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_x$$

$$A_y = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_y$$

دانشگاه خوارزمی





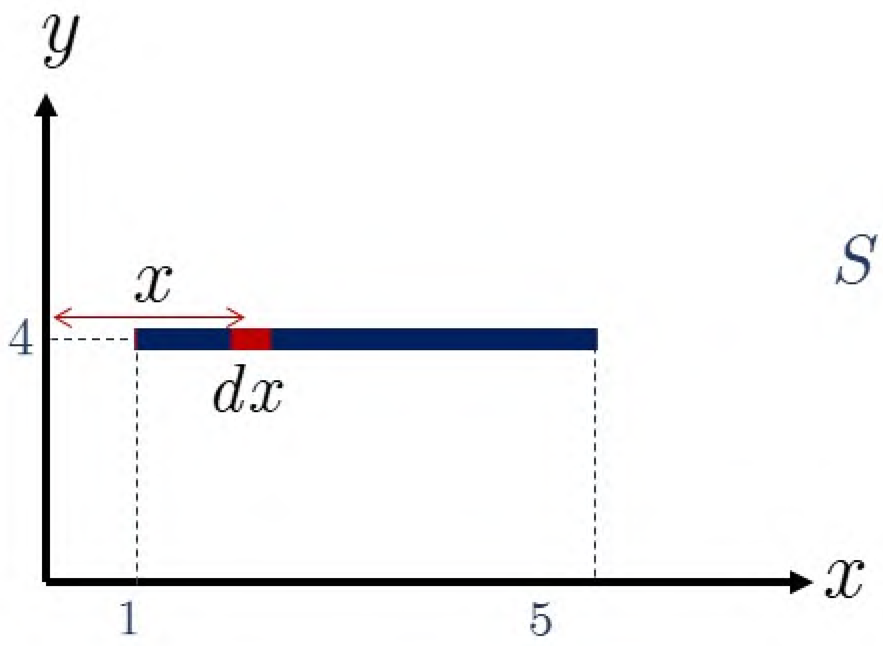
$$\boldsymbol{r} - \boldsymbol{r}' = (x - x')\hat{\boldsymbol{e}}_x + (y - y')\hat{\boldsymbol{e}}_y$$

$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{(x - x')^2 + (y - y')^2}$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = dx\hat{\boldsymbol{e}}_x + dy\hat{\boldsymbol{e}}_y$$

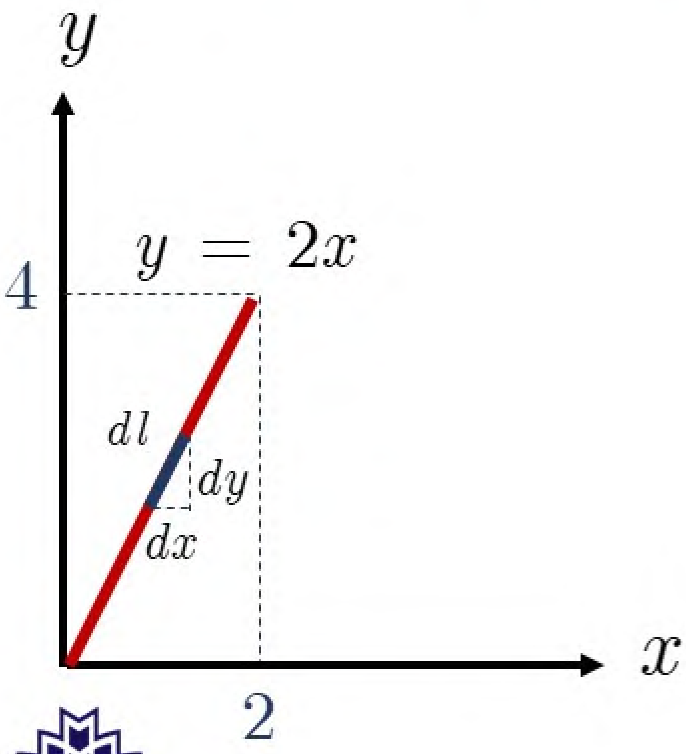
$$da = dxdy$$

دانشگاه خوارزمی



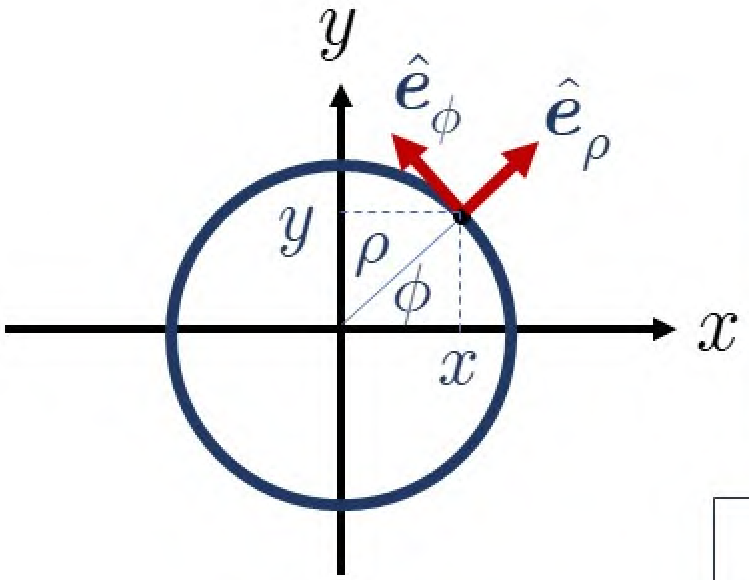
$$S = \int_1^5 dx = x\Big|_1^5 = 5 - 1 = 4$$

$$L = \int_{x=0,y=0}^{x=2,y=4} dl = \int_{x=0,y=0}^{x=2,y=4} \sqrt{dx^2 + dy^2} = \int_{x=0,y=0}^{x=2,y=4} dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$



$$L = \int_{x=0}^{x=2} dx\sqrt{1 + \left(2\right)^2}$$

$$L = \sqrt{5}\int_{x=0}^{x=2} dx = 2\sqrt{5}$$

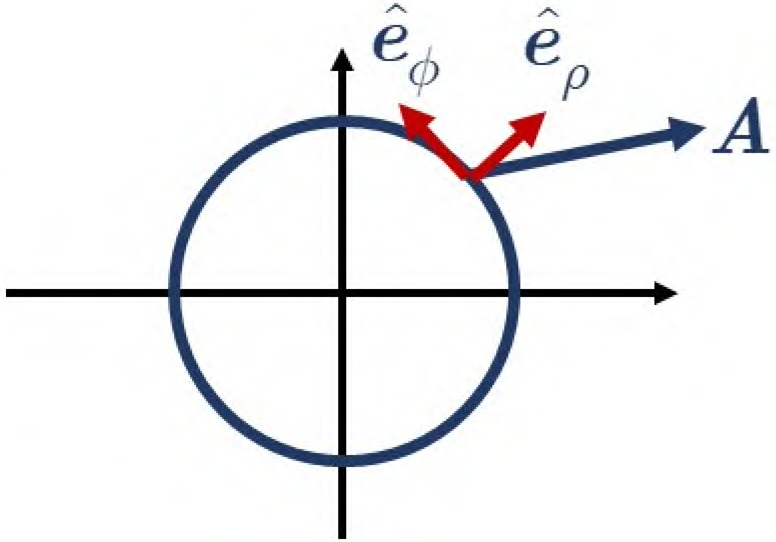


$$\boldsymbol{r} = \rho \hat{\boldsymbol{e}}_\rho$$

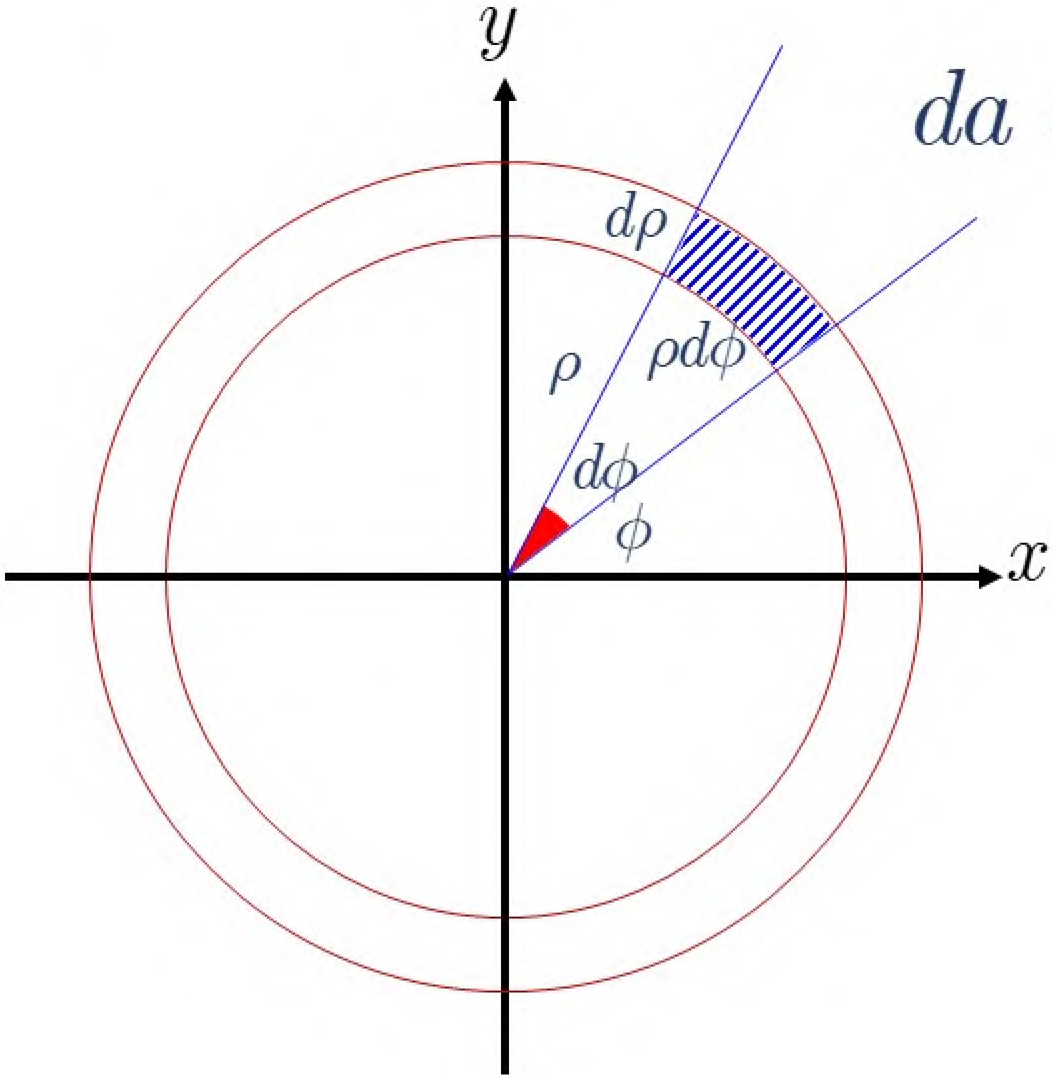
$$x = \rho \cos\phi$$
$$y = \rho \sin\phi$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$

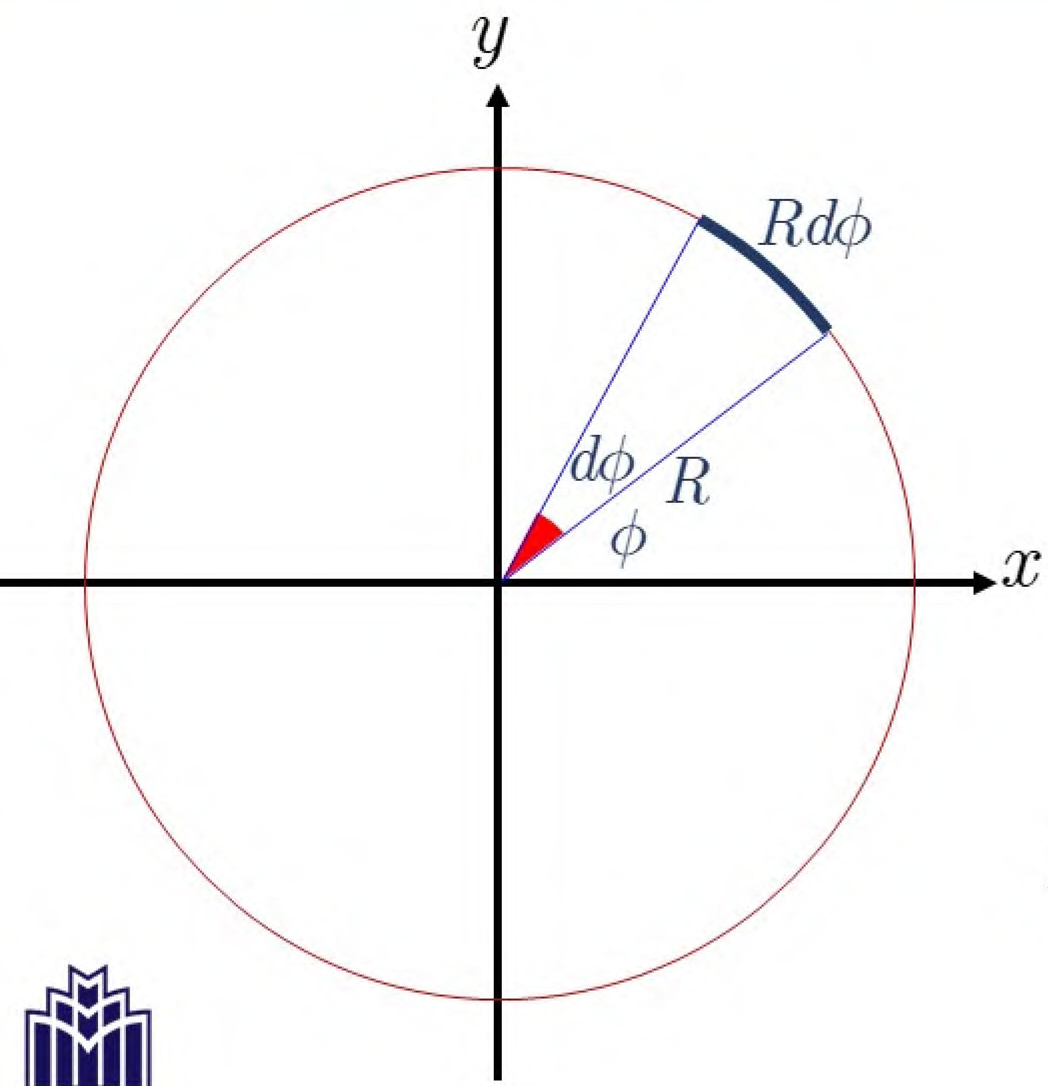
$$\cos\phi = \frac{x}{\sqrt{x^2 + y^2}}$$
$$\sin\phi = \frac{y}{\sqrt{x^2 + y^2}}$$



$$\boldsymbol{A} = A_\rho \hat{\boldsymbol{e}}_\rho + A_\phi \hat{\boldsymbol{e}}_\phi$$

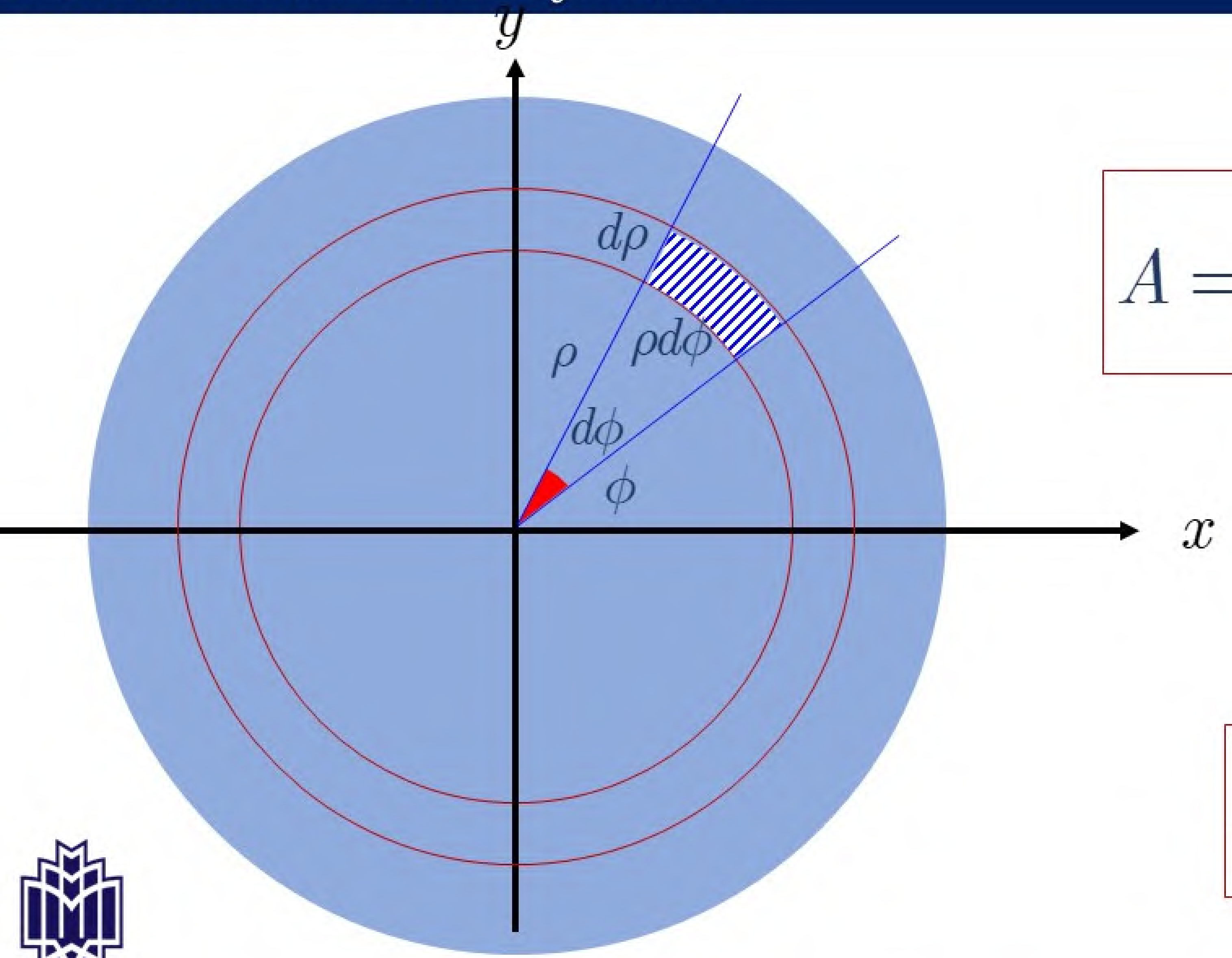


$$da = \rho d\rho d\phi$$



$$dl = Rd\phi$$

$$L = \int dl = R\int_0^{2\pi} d\phi = R\phi\Big|_0^{2\pi} = 2\pi R$$



$$da = \rho d\rho d\phi$$

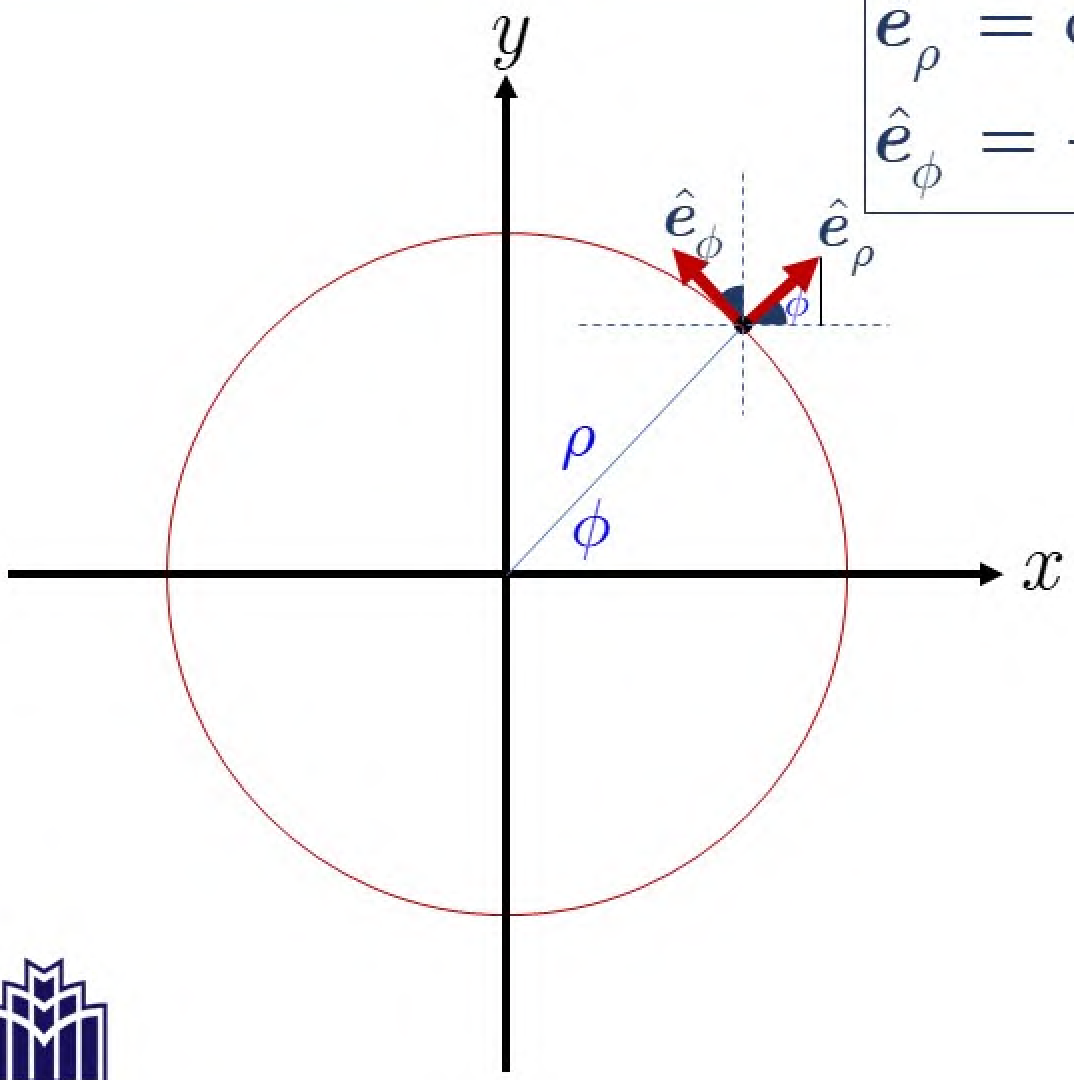
$$A = \int da = \int_0^{2\pi}\int_0^{R} \rho d\rho d\phi$$

$$A = \int_0^{2\pi} d\phi \left[\int_0^{R} \rho d\rho\right]$$

$$A = \left(2\pi\right)\left(\frac{1}{2}R^2\right) = \pi R^2$$

$$\hat{\boldsymbol{e}}_\rho = \cos\phi\,\hat{\boldsymbol{e}}_x + \sin\phi\,\hat{\boldsymbol{e}}_y$$
$$\hat{\boldsymbol{e}}_\phi = -\sin\phi\,\hat{\boldsymbol{e}}_x + \cos\phi\,\hat{\boldsymbol{e}}_y$$

$$\hat{\boldsymbol{e}}_x = \cos\phi\,\hat{\boldsymbol{e}}_\rho - \sin\phi\,\hat{\boldsymbol{e}}_\phi$$
$$\hat{\boldsymbol{e}}_y = \sin\phi\,\hat{\boldsymbol{e}}_\rho + \cos\phi\,\hat{\boldsymbol{e}}_\phi$$



$$A_x = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_x = A_\rho\hat{\boldsymbol{e}}_\rho\cdot\hat{\boldsymbol{e}}_x + A_\phi\hat{\boldsymbol{e}}_\phi\cdot\hat{\boldsymbol{e}}_x$$

$$A_y = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_y = A_\rho\hat{\boldsymbol{e}}_\rho\cdot\hat{\boldsymbol{e}}_y + A_\phi\hat{\boldsymbol{e}}_\phi\cdot\hat{\boldsymbol{e}}_y$$

$$A_x = A_\rho\cos\phi - A_\phi\sin\phi$$
$$A_y = A_\rho\sin\phi + A_\phi\cos\phi$$

$$A_x = A_\rho \cos\phi - A_\phi \sin\phi$$
$$A_y = A_\rho \sin\phi + A_\phi \cos\phi$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\phi \end{pmatrix}$$

$$\begin{pmatrix} A_\rho \\ A_\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & -\frac{y}{\sqrt{x^2+y^2}} \\ \frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\phi \end{pmatrix}$$

شاد و مهربان باشید