Fundamentals of Physics II

Faculty of Physics-Kharazmi University

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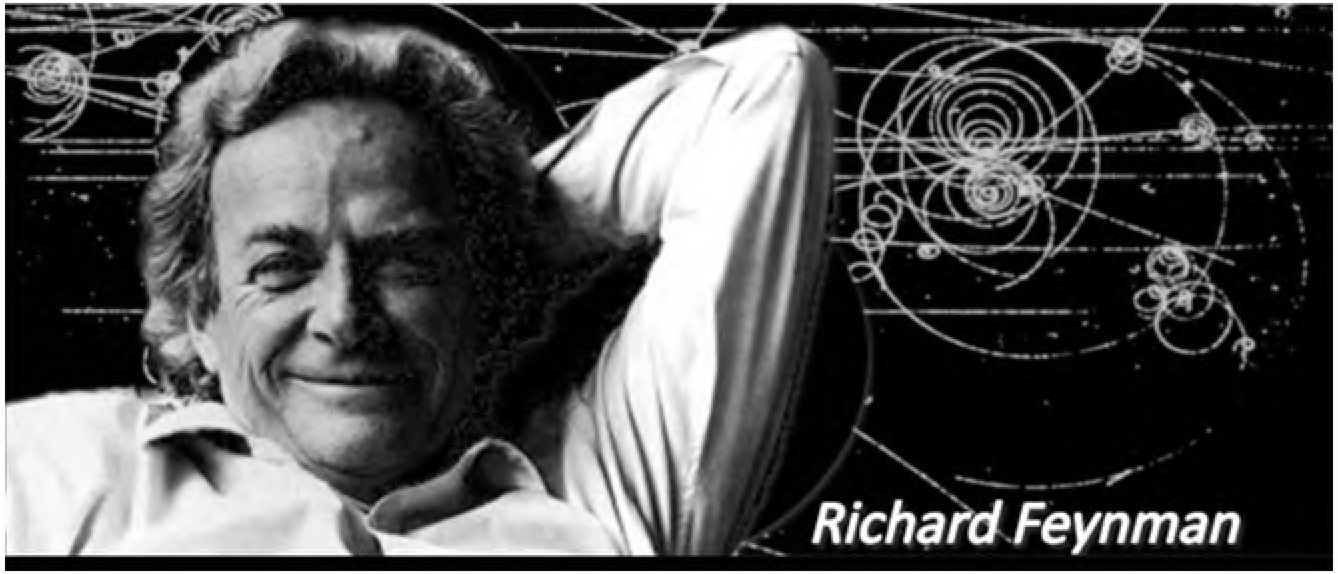


اگر همواره مانند گذشته بیندیشید، همیشه همان چیزهایی را به‌دست می‌آورید که تا کنون کسب کرده‌اید

فاینمن

درس پنجم

دستگاه‌های مختصات- بخش۲

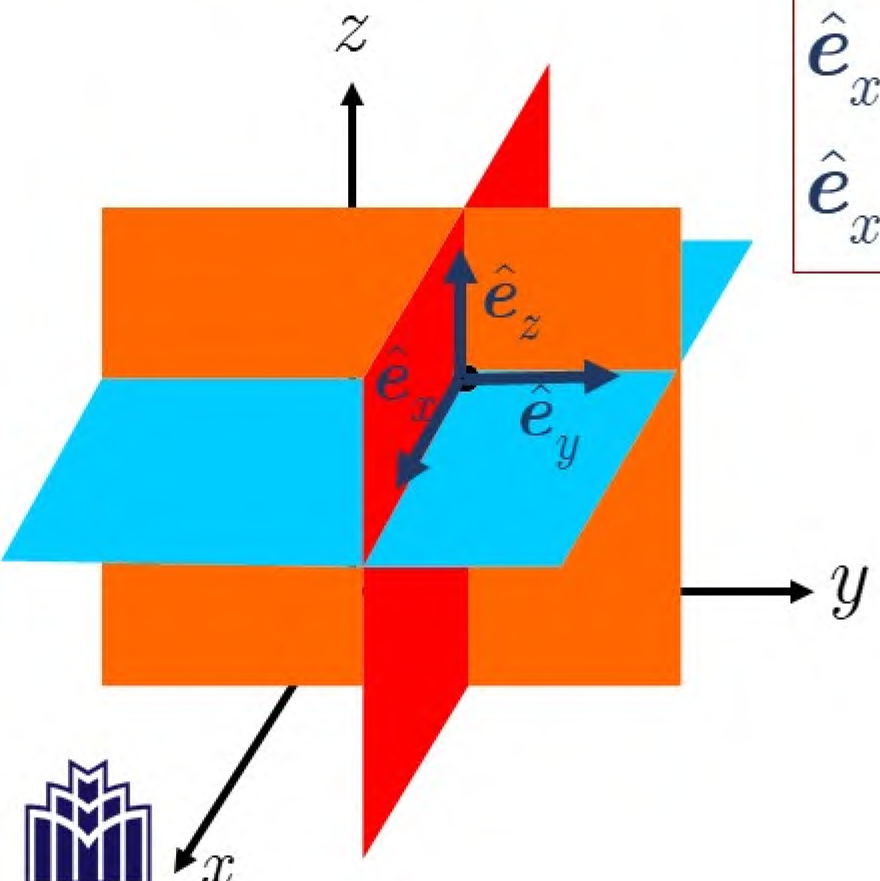
دانشگاه خوارزمی

- ✍ دستگاه مختصات کارتزین سه بُعدی - بردارهای پایه، عنصر طول، سطح و حجم
- ✍ دستگاه مختصات استوانه‌ای - بردارهای پایه، عنصر طول، سطح و حجم
- ✍ دستگاه مختصات کروی - بردارهای پایه، عنصر طول، سطح و حجم
- ✍ رابطه‌ی بین دستگاه مختصات کارتزین و استوانه‌ای
- ✍ رابطه‌ی بین دستگاه مختصات کارتزین و کروی

$$\boldsymbol{r} = x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y + z\hat{\boldsymbol{e}}_z$$

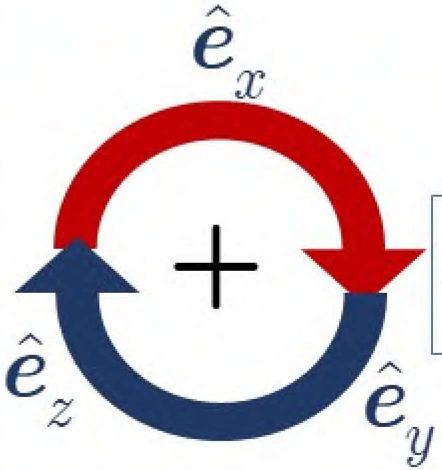
$$\boldsymbol{A} = A_x\hat{\boldsymbol{e}}_x + A_y\hat{\boldsymbol{e}}_y + A_z\hat{\boldsymbol{e}}_z$$

$$A_x = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_x \qquad A_y = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_y \qquad A_z = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_z$$



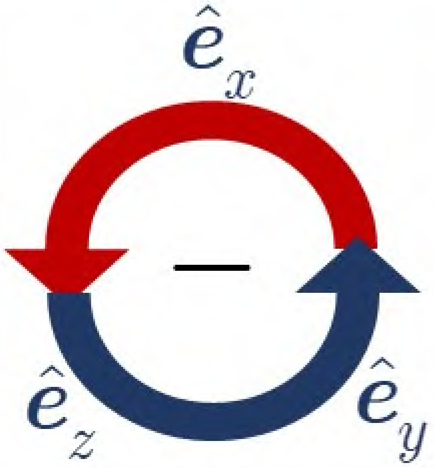
$$\hat{\boldsymbol{e}}_x\cdot\hat{\boldsymbol{e}}_x = \hat{\boldsymbol{e}}_y\cdot\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{e}}_z\cdot\hat{\boldsymbol{e}}_z = 1$$

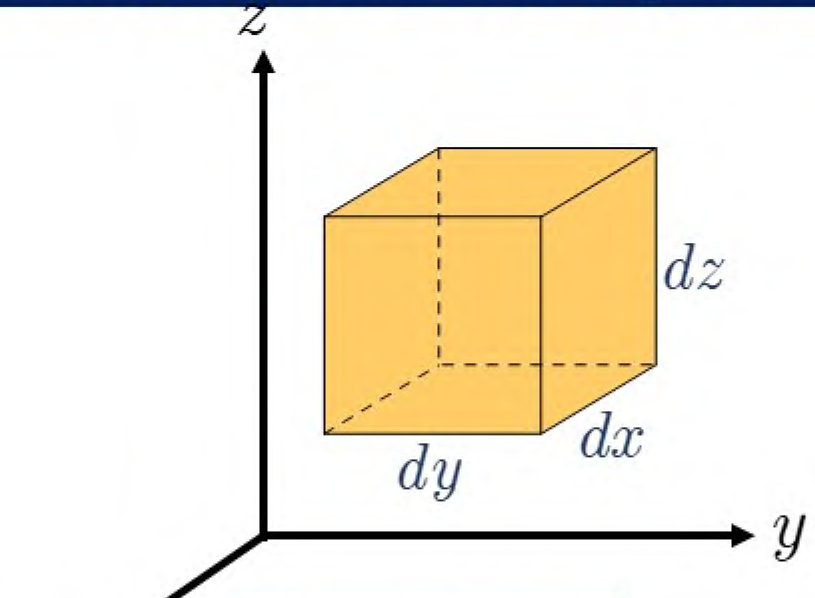
$$\hat{\boldsymbol{e}}_x\cdot\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{e}}_x\cdot\hat{\boldsymbol{e}}_z = \hat{\boldsymbol{e}}_y\cdot\hat{\boldsymbol{e}}_z = 0$$



$$\hat{\boldsymbol{e}}_x\times\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{e}}_z$$

$$\hat{\boldsymbol{e}}_y\times\hat{\boldsymbol{e}}_x = -\hat{\boldsymbol{e}}_z$$

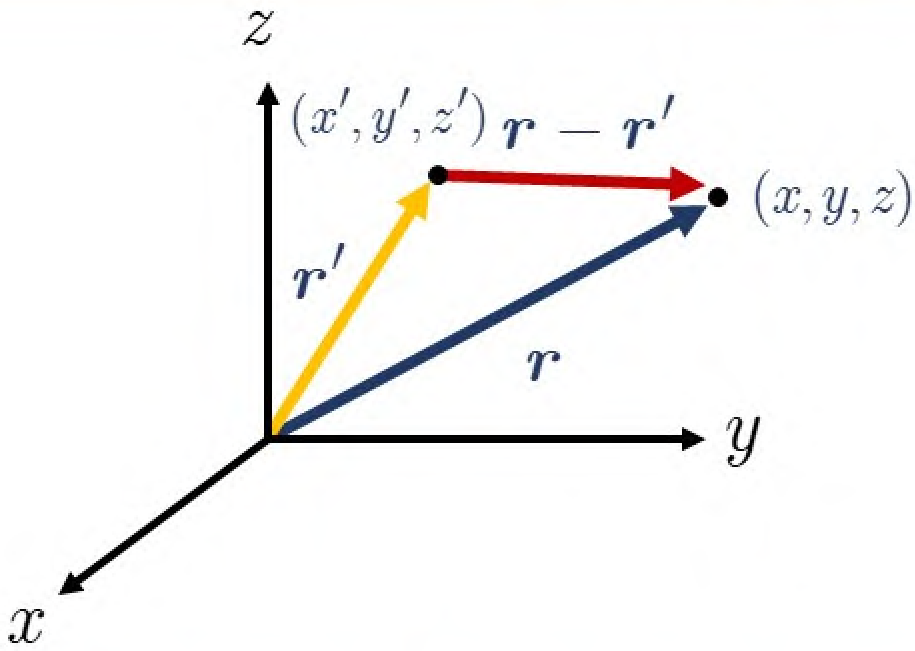




$$dv = dxdydz$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = dx\hat{\boldsymbol{e}}_x + dy\hat{\boldsymbol{e}}_y + dz\hat{\boldsymbol{e}}_z$$

$$\begin{aligned} da_x &= dydz \\ da_y &= dxdz \\ da_z &= dxdy \end{aligned}$$

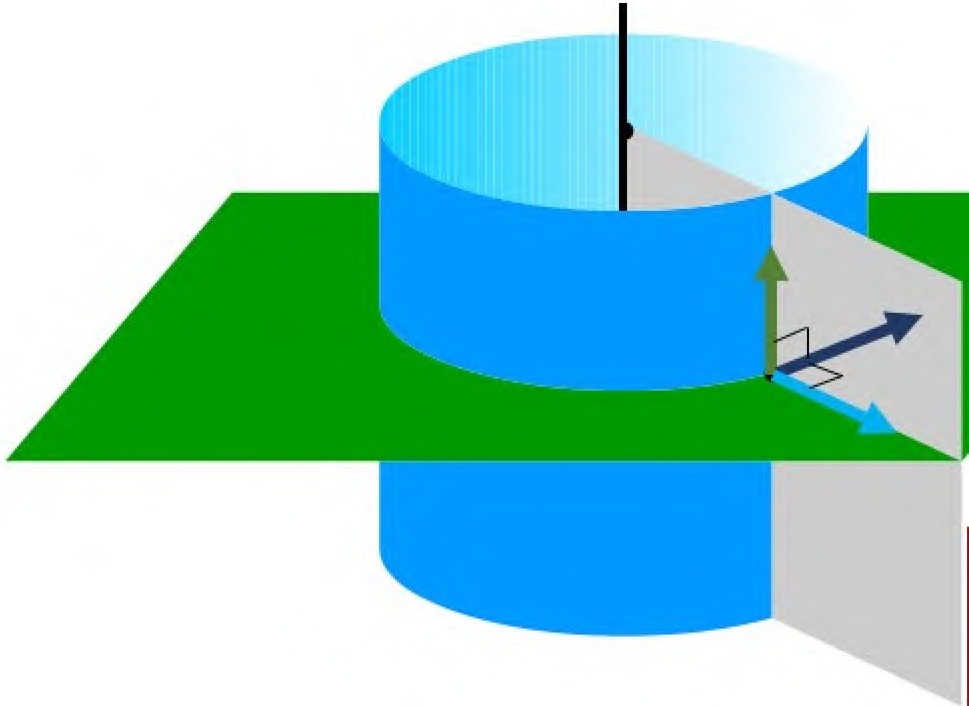


$$\boldsymbol{r} - \boldsymbol{r}' = (x - x')\hat{\boldsymbol{e}}_x + (y - y')\hat{\boldsymbol{e}}_y + (z - z')\hat{\boldsymbol{e}}_z$$

فاصله‌ی دو نقطه:

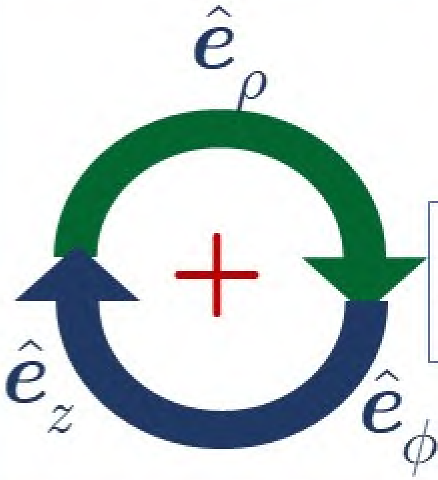
$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

دانشگاه خوارزمی



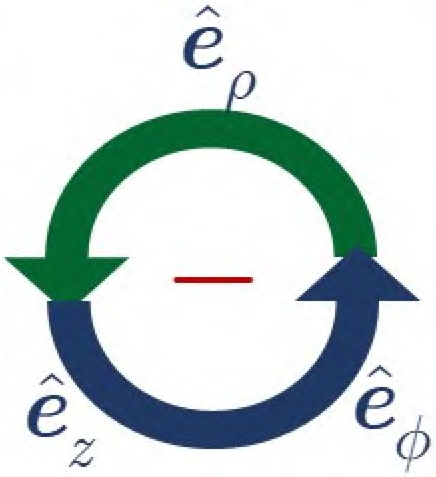
$$\hat{e}_\rho \cdot \hat{e}_\rho = \hat{e}_\phi \cdot \hat{e}_\phi = \hat{e}_z \cdot \hat{e}_z = 1$$

$$\hat{e}_\rho \cdot \hat{e}_z = \hat{e}_\rho \cdot \hat{e}_\phi = \hat{e}_z \cdot \hat{e}_\phi = 0$$

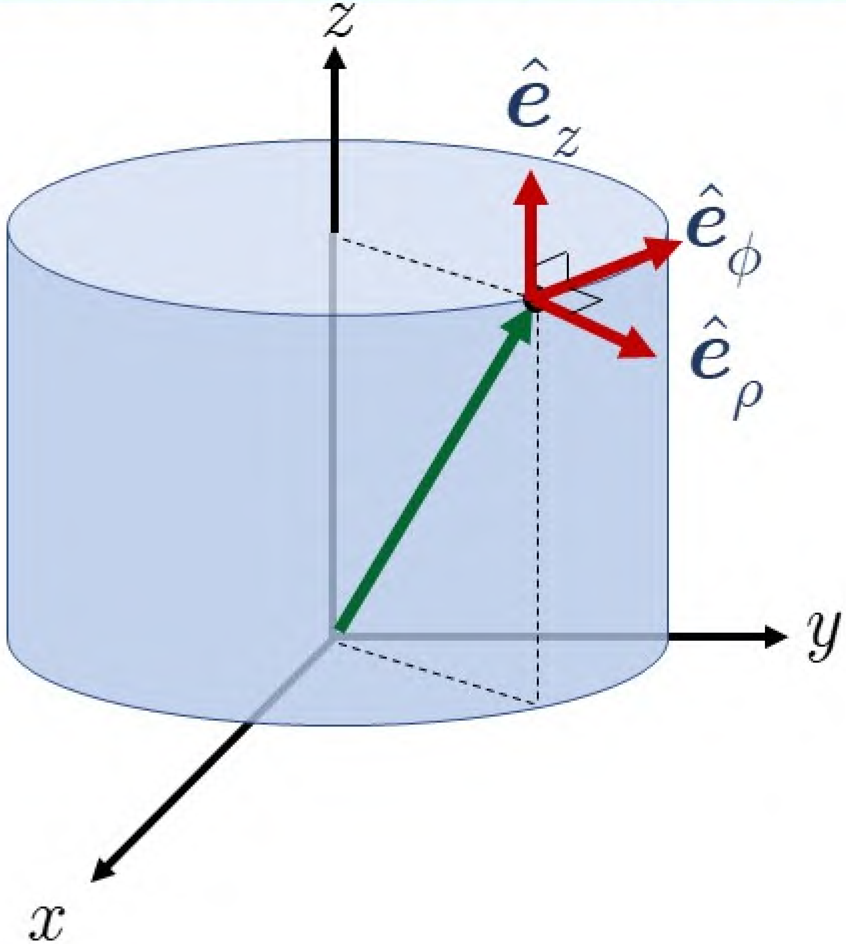


$$\boxed{\hat{e}_\rho \times \hat{e}_\phi = \hat{e}_z}$$

$$\boxed{\hat{e}_\rho \times \hat{e}_z = -\hat{e}_\phi}$$



دانشگاه خوارزمی

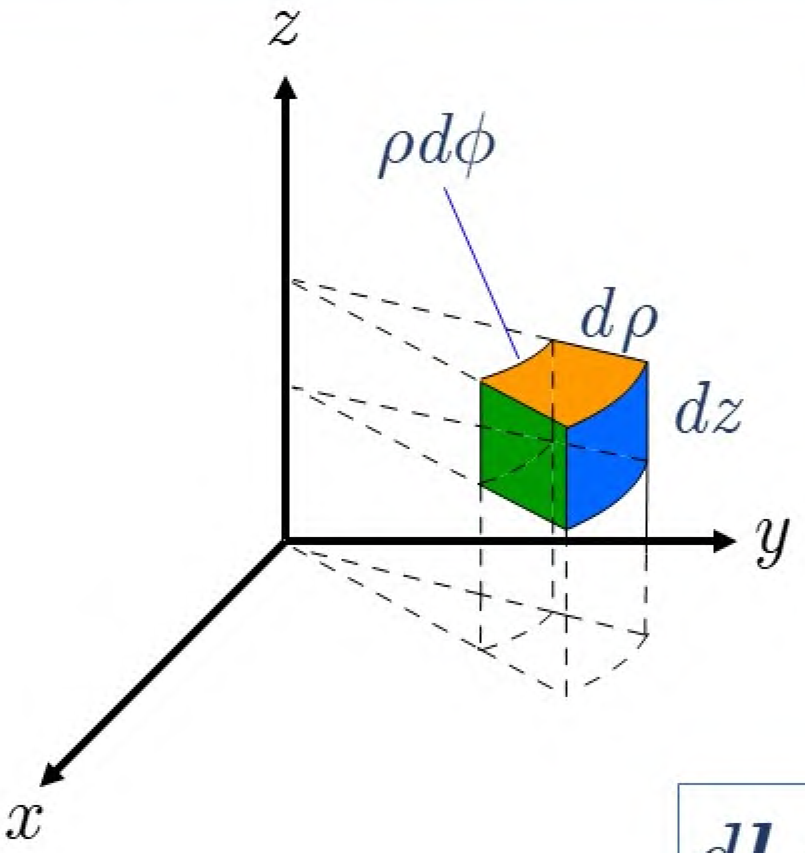


$$\boldsymbol{r} = \rho\hat{\boldsymbol{e}}_\rho + z\hat{\boldsymbol{e}}_z$$

$$|\boldsymbol{r}| = r = \sqrt{\rho^2 + z^2}$$

$$\boldsymbol{A} = A_\rho\hat{\boldsymbol{e}}_\rho + A_\phi\hat{\boldsymbol{e}}_\phi + A_z\hat{\boldsymbol{e}}_z$$

$$|\boldsymbol{A}| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$



$$da_\rho = \rho d\phi dz$$
$$da_\phi = d\rho dz$$
$$da_z = \rho d\rho d\phi$$

$$dv = \rho d\rho d\phi dz$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = d\rho \hat{\boldsymbol{e}}_\rho + \rho d\phi \hat{\boldsymbol{e}}_\phi + dz \hat{\boldsymbol{e}}_z$$

$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi - \phi') + (z - z')^2}$$

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$
$$z = z$$

$$\hat{e}_x = \cos\phi\,\hat{e}_\rho - \sin\phi\,\hat{e}_\phi$$
$$\hat{e}_y = \sin\phi\,\hat{e}_\rho + \cos\phi\,\hat{e}_\phi$$
$$\hat{e}_z = \hat{e}_z$$

$$\hat{e}_\rho = \cos\phi\,\hat{e}_x + \sin\phi\,\hat{e}_y$$
$$\hat{e}_\phi = -\sin\phi\,\hat{e}_x + \cos\phi\,\hat{e}_y$$
$$\hat{e}_z = \hat{e}_z$$

دانشگاه خوارزمی

$$\boldsymbol{A} = A_\rho \hat{\boldsymbol{e}}_\rho + A_\phi \hat{\boldsymbol{e}}_\phi + A_z \hat{\boldsymbol{e}}_z$$

$$\begin{aligned}
A_x &= \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_x = A_\rho\hat{\boldsymbol{e}}_\rho\cdot\hat{\boldsymbol{e}}_x + A_\phi\hat{\boldsymbol{e}}_\phi\cdot\hat{\boldsymbol{e}}_x + A_z\hat{\boldsymbol{e}}_z\cdot\hat{\boldsymbol{e}}_x\\
A_y &= \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_y = A_\rho\hat{\boldsymbol{e}}_\rho\cdot\hat{\boldsymbol{e}}_y + A_\phi\hat{\boldsymbol{e}}_\phi\cdot\hat{\boldsymbol{e}}_y + A_z\hat{\boldsymbol{e}}_z\cdot\hat{\boldsymbol{e}}_y\\
A_z &= \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_z = A_\rho\hat{\boldsymbol{e}}_\rho\cdot\hat{\boldsymbol{e}}_z + A_\phi\hat{\boldsymbol{e}}_\phi\cdot\hat{\boldsymbol{e}}_z + A_z\hat{\boldsymbol{e}}_z\cdot\hat{\boldsymbol{e}}_z
\end{aligned}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{-y}{\sqrt{x^2+y^2}} & 0 \\ \frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix}$$

$$\boxed{\begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}}$$

بردار $A = \hat{e}_x + x\hat{e}_y + (x+y)\hat{e}_z$ در دستگاه مختصات کارتزین داده شده است. مؤلفه‌های این بردار را در دستگاه مختصات استوانه‌ای در نقطه‌ی (-1,2,3) بیابید.

$$\rho = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 2^2} = 2.24$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2}{-1} = 116.57^\circ$$

$$z = 3$$

$$\begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ x+y \end{pmatrix}$$

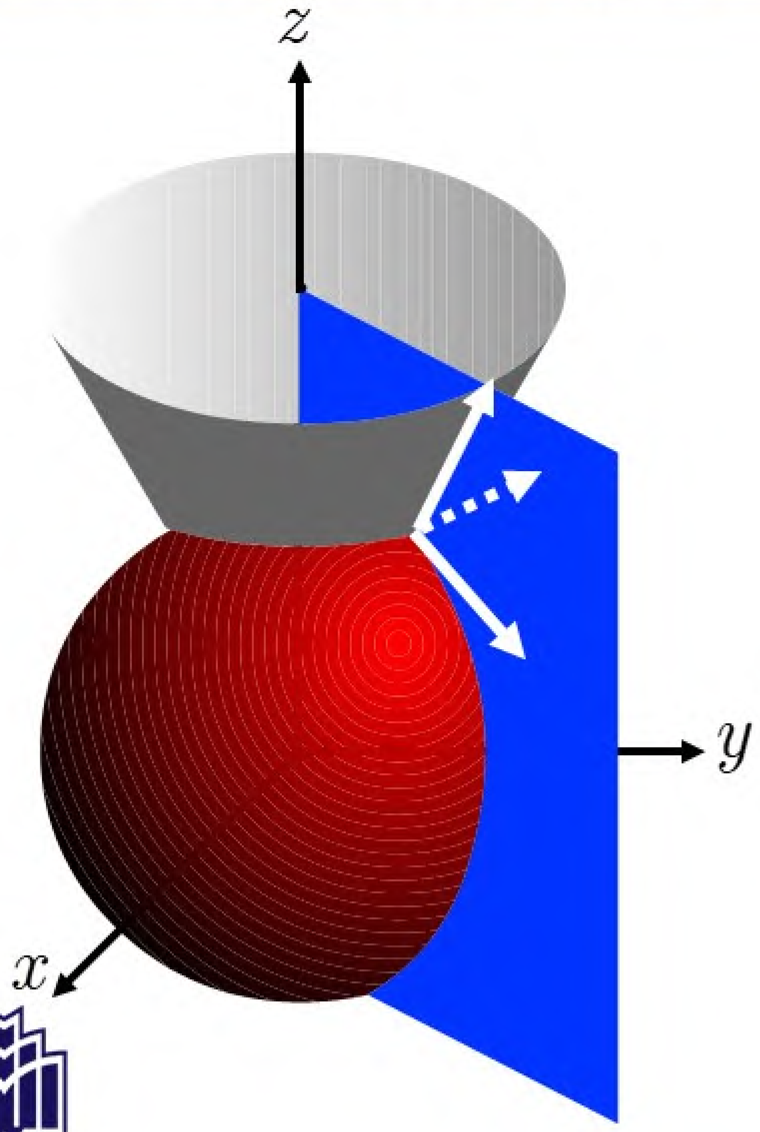
$$\begin{aligned} A_\rho &= \cos\phi + x\sin\phi \\ A_\phi &= -\sin\phi + x\cos\phi \\ A_z &= x + y \end{aligned}$$

$$\begin{aligned} A_\rho &= \cos\phi + \rho\cos\phi\sin\phi = -1.34 \\ A_\phi &= -\sin\phi + \rho\cos\phi\cos\phi = -0.45 \\ A_z &= \rho\cos\phi + \rho\sin\phi = 1 \end{aligned}$$

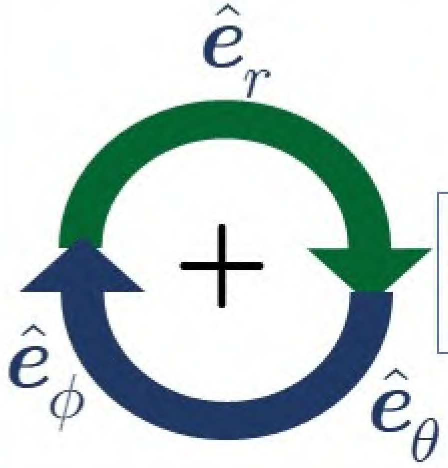
بنابر این در نقطه ی (-1,2,3) بردار فوق برابر است

$$\left(A_\rho, A_\phi, A_z\right) = \left(-1.34, -0.45, 1\right)$$

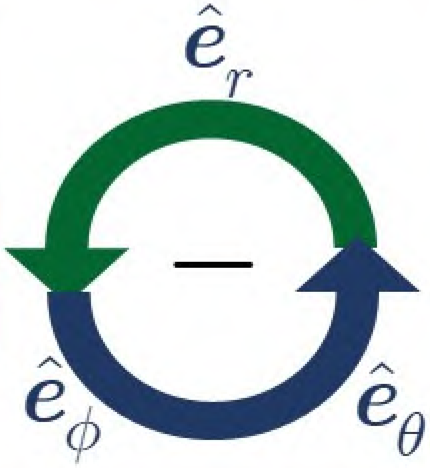
دانشگاه خوارزمی

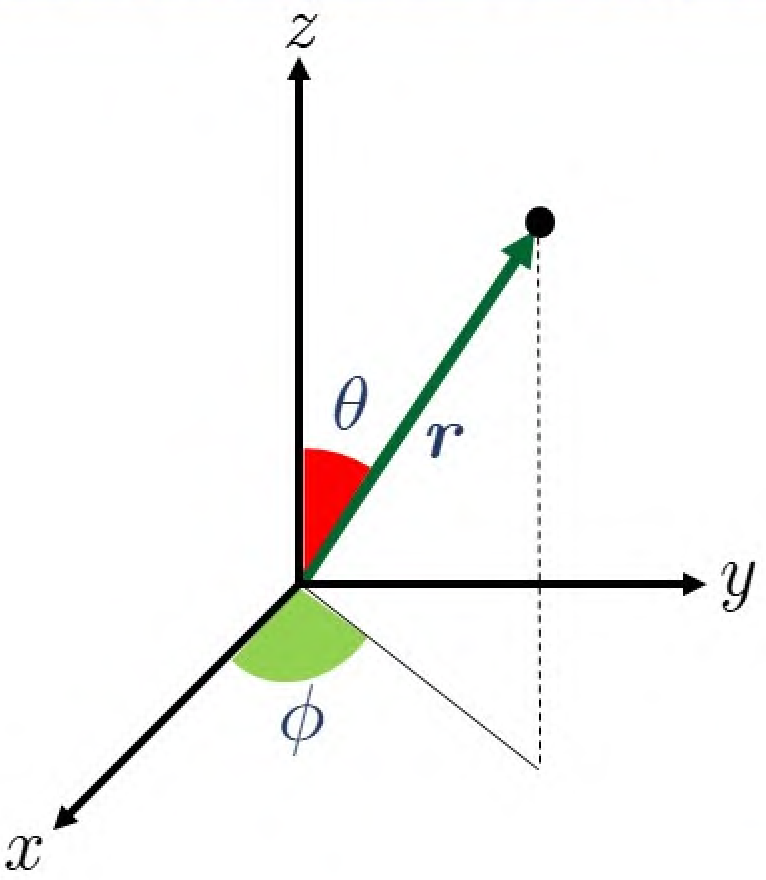


$$\hat{\boldsymbol{e}}_r \cdot \hat{\boldsymbol{e}}_r = \hat{\boldsymbol{e}}_\theta \cdot \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_\phi = 1$$
$$\hat{\boldsymbol{e}}_r \cdot \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_r \cdot \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_\theta \cdot \hat{\boldsymbol{e}}_\phi = 0$$



$$\hat{\boldsymbol{e}}_r \times \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_\phi$$
$$\hat{\boldsymbol{e}}_r \times \hat{\boldsymbol{e}}_\phi = -\hat{\boldsymbol{e}}_\theta$$



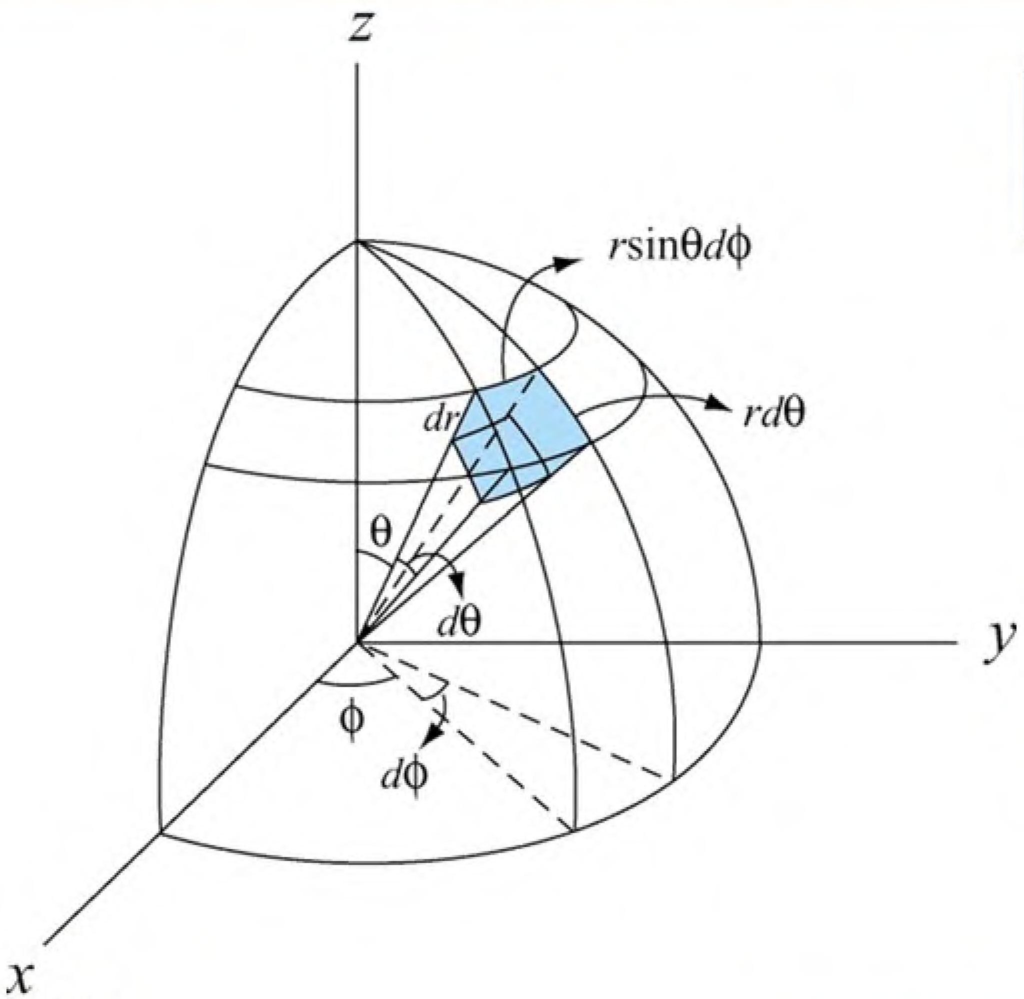


$$\boldsymbol{r} = r\hat{\boldsymbol{e}}_r$$

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}}$$

$$\boxed{\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \theta &= \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}} \\ \phi &= \tan^{-1}\frac{y}{x} \end{aligned}}$$

$$\boxed{d = \left|\boldsymbol{r}-\boldsymbol{r}'\right| = \sqrt{r^2+r'^2-2rr'\cos\theta\cos\theta'-2rr'\sin\theta\sin\theta'\cos(\phi-\phi')}}$$



$$dv = r^2 \sin\theta dr d\theta d\phi$$

$$da_r = r^2 \sin\theta d\theta d\phi$$
$$da_\theta = r \sin\theta dr d\phi$$
$$da_\phi = r d\theta dr$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = dr\hat{\boldsymbol{e}}_r + rd\theta\hat{\boldsymbol{e}}_\theta + r\sin\theta d\phi\hat{\boldsymbol{e}}_\phi$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix}$$

دانشگاه خوارزمی

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

شاد و مهربان باشید