## Broblems

| so | Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| SSM | Worked-out solution available in Student Solutions Manual | WWW | Worked-out solution is at |  |
| -- http://www.wiley.com/college/halliday |  |  |  |  |

## Module 28-1 Magnetic Fields and the Definition of $\vec{B}$

-1 SSM ILW A proton traveling at $23.0^{\circ}$ with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of $6.50 \times 10^{-17} \mathrm{~N}$. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts.
-2 A particle of mass 10 g and charge $80 \mu \mathrm{C}$ moves through a uniform magnetic field, in a region where the free-fall acceleration is $-9.8 \hat{\mathrm{j}} \mathrm{m} / \mathrm{s}^{2}$. The velocity of the particle is a constant $20 \hat{\mathrm{i}} \mathrm{km} / \mathrm{s}$, which is perpendicular to the magnetic field. What, then, is the magnetic field?
-3 An electron that has an instantaneous velocity of

$$
\vec{v}=\left(2.0 \times 10^{6} \mathrm{~m} / \mathrm{s}\right) \hat{\mathrm{i}}+\left(3.0 \times 10^{6} \mathrm{~m} / \mathrm{s}\right) \hat{\mathrm{j}}
$$

is moving through the uniform magnetic field $\vec{B}=(0.030 \mathrm{~T}) \hat{\mathrm{i}}-$ $(0.15 \mathrm{~T}) \hat{\mathrm{j}}$. (a) Find the force on the electron due to the magnetic field. (b) Repeat your calculation for a proton having the same velocity.
-4 An alpha particle travels at a velocity $\vec{v}$ of magnitude $550 \mathrm{~m} / \mathrm{s}$ through a uniform magnetic field $\vec{B}$ of magnitude 0.045 T . (An alpha particle has a charge of $+3.2 \times 10^{-19} \mathrm{C}$ and a mass of $6.6 \times 10^{-27} \mathrm{~kg}$.) The angle between $\vec{v}$ and $\vec{B}$ is $52^{\circ}$. What is the magnitude of (a) the force $\vec{F}_{B}$ acting on the particle due to the field and (b) the acceleration of the particle due to $\vec{F}_{B}$ ? (c) Does the speed of the particle increase, decrease, or remain the same?
$\bullet$-5 An electron moves through a uniform magnetic field given by $\vec{B}=B_{x} \hat{i}+\left(3.0 B_{x}\right) \hat{\mathrm{j}}$. At a particular instant, the electron has velocity $\vec{v}=(2.0 \hat{\mathrm{i}}+4.0 \hat{\mathrm{j}}) \mathrm{m} / \mathrm{s}$ and the magnetic force acting on it is $\left(6.4 \times 10^{-19} \mathrm{~N}\right) \hat{\mathrm{k}}$. Find $B_{x}$.
$\bullet \bullet 6$ A proton moves through a uniform magnetic field given by $\vec{B}=(10 \hat{\mathrm{i}}-20 \hat{\mathrm{j}}+30 \hat{\mathrm{k}}) \mathrm{mT}$. At time $t_{1}$, the proton has a velocity given by $\vec{v}_{\vec{\prime}}=v_{x} \hat{i}+v_{y} \hat{j}+(2.0 \mathrm{~km} / \mathrm{s}) \hat{\mathrm{k}}$ and the magnetic force on the proton is $\vec{F}_{B}=\left(4.0 \times 10^{-17} \mathrm{~N}\right) \hat{\mathrm{i}}+\left(2.0 \times 10^{-17} \mathrm{~N}\right) \hat{\mathrm{j}}$. At that instant, what are (a) $v_{x}$ and (b) $\nu_{y}$ ?

## Module 28-2 Crossed Fields: Discovery of the Electron

-7 An electron has an initial velocity of $(12.0 \hat{j}+15.0 \hat{k}) \mathrm{km} / \mathrm{s}$ and a constant acceleration of $\left(2.00 \times 10^{12} \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathrm{i}}$ in a region in which uniform electric and magnetic fields are present. If $\vec{B}=(400 \mu \mathrm{~T}) \hat{\mathrm{i}}$, find the electric field $\vec{E}$.
-8 An electric field of $1.50 \mathrm{kV} / \mathrm{m}$ and a perpendicular magnetic field of 0.400 T act on a moving electron to produce no net force. What is the electron's speed?
$\bullet 9$ ILW In Fig. 28-32, an electron accelerated from rest through potential difference $V_{1}=1.00 \mathrm{kV}$ enters the gap between two parallel plates having separation $d=20.0 \mathrm{~mm}$ and potential difference


Figure 28-32 Problem 9.
$V_{2}=100 \mathrm{~V}$. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. In unit-vector notation, what uniform magnetic field allows the electron to travel in a straight line in the gap?

- 10 A proton travels through uniform magnetic and electric fields. The magnetic field is $\vec{B}=-2.50 \hat{\mathrm{i}} \mathrm{mT}$. At one instant the velocity of the proton is $\vec{v}=2000 \hat{\mathrm{j}} \mathrm{m} / \mathrm{s}$. At that instant and in unit-vector notation, what is the net force acting on the proton if the electric field is (a) $4.00 \hat{\mathrm{k}} \mathrm{V} / \mathrm{m}$, (b) $-4.00 \hat{\mathrm{k}} \mathrm{V} / \mathrm{m}$, and (c) $4.00 \hat{\mathrm{i}} \mathrm{V} / \mathrm{m}$ ?
- 11 (60 An ion source is producing ${ }^{6} \mathrm{Li}$ ions, which have charge $+e$ and mass $9.99 \times 10^{-27} \mathrm{~kg}$. The ions are accelerated by a potential difference of 10 kV and pass horizontally into a region in which there is a uniform vertical magnetic field of magnitude $B=1.2 \mathrm{~T}$. Calculate the strength of the electric field, to be set up over the same region, that will allow the ${ }^{6} \mathrm{Li}$ ions to pass through without any deflection.
$\cdots 12$ (so At time $t_{1}$, an electron is sent along the positive direction of an $x$ axis, through both an electric $\xrightarrow{\text { field }} \vec{E}$ and a magnetic field $\vec{B}$, with $\vec{E}$ directed parallel to the $y$ axis. Figure 28-33 gives the $y$ component $F_{\text {net,y }}$ of the net force on the electron due to the two fields, as a function of the electron's speed $v$ at time $t_{1}$. The scale of the velocity axis is set by $v_{s}=100.0 \mathrm{~m} / \mathrm{s}$. The $x$ and $z$ components of the net force are zero


Figure 28-33 Problem 12. at $t_{1}$. Assuming $B_{x}=0$, find (a) the magnitude $E$ and (b) $\vec{B}$ in unitvector notation.

## Module 28-3 Crossed Fields: The Hall Effect

-13 A strip of copper $150 \mu \mathrm{~m}$ thick and 4.5 mm wide is placed in a uniform magnetic field $\vec{B}$ of magnitude 0.65 T , with $\vec{B}$ perpendicular to the strip. A current $i=23 \mathrm{~A}$ is then sent through the strip such that a Hall potential difference $V$ appears across the width of the strip. Calculate $V$. (The number of charge carriers per unit volume for copper is $8.47 \times 10^{28}$ electrons $/ \mathrm{m}^{3}$.)
-14 A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity $\vec{v}$ through a uniform magnetic field $B=1.20 \mathrm{mT}$ directed perpendicular to the strip, as shown in Fig. 28-34. A potential difference of $3.90 \mu \mathrm{~V}$ is measured between points $x$ and $y$ across the strip. Calculate the speed $v$.
$\bullet 15$ (so A conducting rectangular solid of dimensions $d_{x}=5.00 \mathrm{~m}$, $d_{y}=3.00 \mathrm{~m}$, and $d_{z}=2.00 \mathrm{~m}$ moves with a constant velocity $\vec{v}=(20.0 \mathrm{~m} / \mathrm{s}) \hat{\mathrm{i}}$ through a uniform
magnetic field $\vec{B}=(30.0 \mathrm{mT}) \hat{\mathrm{j}}$ (Fig. 28-35). What are the resulting (a) electric field within the solid, in unit-vector notation, and (b) potential difference across the solid?
-••16
Figure $28-35$ shows a metallic block, with its faces parallel to coordinate axes. The block is in a uniform magnetic field of magnitude 0.020 T . One edge length of the block is 25 cm ; the block is not drawn to scale. The block is moved at $3.0 \mathrm{~m} / \mathrm{s}$ parallel to each axis, in turn, and the resulting potential difference $V$ that appears across the block


Figure 28-35 Problems 15 and 16. is measured. With the motion parallel to the $y$ axis, $V=12 \mathrm{mV}$; with the motion parallel to the $z$ axis, $V=18 \mathrm{mV}$; with the motion parallel to the $x$ axis, $V=0$. What are the block lengths (a) $d_{x}$, (b) $d_{y}$, and (c) $d_{z}$ ?

## Module 28-4 A Circulating Charged Particle

-17 An alpha particle can be produced in certain radioactive decays of nuclei and consists of two protons and two neutrons. The particle has a charge of $q=+2 e$ and a mass of 4.00 u , where u is the atomic mass unit, with $1 \mathrm{u}=1.661 \times 10^{-27} \mathrm{~kg}$. Suppose an alpha particle travels in a circular path of radius 4.50 cm in a uniform magnetic field with $B=1.20 \mathrm{~T}$. Calculate (a) its speed, (b) its period of revolution, (c) its kinetic energy, and (d) the potential difference through which it would have to be accelerated to achieve this energy.
-18 © In Fig. 28-36, a particle moves along a circle in a region of uniform magnetic field of magnitude $B=4.00 \mathrm{mT}$. The particle is either a proton or an electron (you must decide which). It experiences a magnetic force of magnitude $3.20 \times 10^{-15} \mathrm{~N}$. What are (a) the particle's speed, (b) the radius of the circle, and (c) the period of


Figure 28-36 Problem 18. the motion?
-19 A certain particle is sent into a uniform magnetic field, with the particle's velocity vector perpendicular to the direction of the field. Figure 28-37 gives the period $T$ of the particle's motion versus the inverse of the field magnitude $B$. The vertical axis scale is set by $T_{s}=40.0 \mathrm{~ns}$, and the horizontal axis scale is set by $B_{s}^{-1}=5.0 \mathrm{~T}^{-1}$. What is the ratio $m / q$ of the particle's mass to the magnitude of its charge?


Figure 28-37 Problem 19.
-20 An electron is accelerated from rest through potential difference $V$ and then enters a region of uniform magnetic field, where it
undergoes uniform circular motion. Figure 28-38 gives the radius $r$ of that motion versus $V^{1 / 2}$. The vertical axis scale is set by $r_{s}=3.0 \mathrm{~mm}$, and the horizontal axis scale is set by $V_{s}^{1 / 2}=40.0 \mathrm{~V}^{1 / 2}$. What is the magnitude of the magnetic field?
-21 SSM An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic


Figure 28-38 Problem 20. field. The orbit radius is 25.0 cm . Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion.
-22 In a nuclear experiment a proton with kinetic energy 1.0 MeV moves in a circular path in a uniform magnetic field. What energy must (a) an alpha particle ( $q=+2 e, m=4.0 \mathrm{u}$ ) and (b) a deuteron ( $q=+e, m=2.0 \mathrm{u}$ ) have if they are to circulate in the same circular path?
-23 What uniform magnetic field, applied perpendicular to a beam of electrons moving at $1.30 \times 10^{6} \mathrm{~m} / \mathrm{s}$, is required to make the electrons travel in a circular arc of radius 0.350 m ?
-24 An electron is accelerated from rest by a potential difference of 350 V . It then enters a uniform magnetic field of magnitude 200 mT with its velocity perpendicular to the field. Calculate (a) the speed of the electron and (b) the radius of its path in the magnetic field.
-25 (a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude $35.0 \mu \mathrm{~T}$. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field.

- 26 In Fig. 28-39, a charged particle moves into a region of uniform magnetic field $\vec{B}$, goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide


Figure 28-39 Problem 26. which). It spends 130 ns in the region. (a) What is the magnitude of $\vec{B}$ ? (b) If the particle is sent back through the magnetic field (along the same initial path) but with 2.00 times its previous kinetic energy, how much time does it spend in the field during this trip?
$\bullet 27$ A mass spectrometer (Fig. 28-12) is used to separate uranium ions of mass $3.92 \times 10^{-25} \mathrm{~kg}$ and charge $3.20 \times 10^{-19} \mathrm{C}$ from related species. The ions are accelerated through a potential difference of 100 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 1.00 m . After traveling through $180^{\circ}$ and passing through a slit of width 1.00 mm and height 1.00 cm , they are collected in a cup. (a) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is used to separate out 100 mg of material per hour, calculate (b) the current of the desired ions in the machine and (c) the thermal energy produced in the cup in 1.00 h .

- 28 A particle undergoes uniform circular motion of radius $26.1 \mu \mathrm{~m}$ in a uniform magnetic field. The magnetic force on the particle has a magnitude of $1.60 \times 10^{-17} \mathrm{~N}$. What is the kinetic energy of the particle?
-•29 An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T . The pitch of the path is $6.00 \mu \mathrm{~m}$, and the
magnitude of the magnetic force on the electron is $2.00 \times 10^{-15} \mathrm{~N}$. What is the electron's speed?
-•30 60 In Fig. 28-40, an electron with an initial kinetic energy of 4.0 keV enters region 1 at time $t=0$. That region contains a uniform magnetic field directed into the page, with magnitude 0.010 T . The electron goes through a half-circle and then exits region 1 , headed toward region 2 across a gap of 25.0 cm . There is an electric potential difference $\Delta V=2000 \mathrm{~V}$ across the gap, with a polarity such that the electron's speed increases uniformly as it traverses the gap. Region 2 contains a uniform magnetic field directed out of the page, with magnitude 0.020 T. The electron goes through a half-circle and then leaves region 2. At what time $t$ does it leave?
- 31 A particular type of fundamental particle decays by transforming into an electron $\mathrm{e}^{-}$and a positron $\mathrm{e}^{+}$. Suppose the decaying particle is at rest in a uniform magnetic field $\vec{B}$ of magnitude 3.53 mT and the $\mathrm{e}^{-}$and $\mathrm{e}^{+}$move away from the decay point in paths lying in a plane perpendicular to $\vec{B}$. How long after the decay do the $\mathrm{e}^{-}$and $\mathrm{e}^{+}$collide?
- 32 A source injects an electron of speed $v=1.5 \times 10^{7} \mathrm{~m} / \mathrm{s}$ into a uniform magnetic field of magnitude $B=1.0 \times 10^{-3} \mathrm{~T}$. The velocity of the electron makes an angle $\theta=10^{\circ}$ with the direction of the magnetic field. Find the distance $d$ from the point of injection at which the electron next crosses the field line that passes through the injection point.
$\bullet$-33 SSM www A positron with kinetic energy 2.00 keV is projected into a uniform magnetic field $\vec{B}$ of magnitude 0.100 T , with its velocity vector making an angle of $89.0^{\circ}$ with $\vec{B}$. Find (a) the period, (b) the pitch $p$, and (c) the radius $r$ of its helical path.
-•34 An electron follows a helical path in a uniform magnetic field given by $\vec{B}=(20 \hat{\mathrm{i}}-50 \hat{\mathrm{j}}-30 \hat{\mathrm{k}}) \mathrm{mT}$. At time $t=0$, the electron's velocity is given by $\vec{v}=(20 \hat{\mathrm{i}}-30 \hat{\mathrm{j}}+50 \hat{\mathrm{k}}) \mathrm{m} / \mathrm{s}$. (a) What is the angle $\phi$ between $\vec{v}$ and $\vec{B}$ ? The electron's velocity changes with time. Do (b) its speed and (c) the angle $\phi$ change with time? (d) What is the radius of the helical path?


## Module 28-5 Cyclotrons and Synchrotrons

-•35 A proton circulates in a cyclotron, beginning approximately at rest at the center. Whenever it passes through the gap between dees, the electric potential difference between the dees is 200 V . (a) By how much does its kinetic energy increase with each passage through the gap? (b) What is its kinetic energy as it completes 100 passes through the gap? Let $r_{100}$ be the radius of the proton's circular path as it completes those 100 passes and enters a dee, and let $r_{101}$ be its next radius, as it enters a dee the next time. (c) By what percentage does the radius increase when it changes from $r_{100}$ to $r_{101}$ ? That is, what is

$$
\text { percentage increase }=\frac{r_{101}-r_{100}}{r_{100}} 100 \% ?
$$

-•36 A cyclotron with dee radius 53.0 cm is operated at an oscillator frequency of 12.0 MHz to accelerate protons. (a) What magnitude $B$ of magnetic field is required to achieve resonance? (b) At that field magnitude, what is the kinetic energy of a proton emerging from the cyclotron? Suppose, instead, that $B=1.57$ T. (c) What oscillator frequency is required to achieve resonance now? (d) At that frequency, what is the kinetic energy of an emerging proton?
-•37 Estimate the total path length traveled by a deuteron in a cyclotron of radius 53 cm and operating frequency 12 MHz during the (entire) acceleration process. Assume that the accelerating potential between the dees is 80 kV .

- 38 In a certain cyclotron a proton moves in a circle of radius 0.500 m . The magnitude of the magnetic field is 1.20 T . (a) What is the oscillator frequency? (b) What is the kinetic energy of the proton, in electron-volts?


## Module 28-6 Magnetic Force on a Current-Carrying Wire

-39 SSM A horizontal power line carries a current of 5000 A from south to north. Earth's magnetic field $(60.0 \mu \mathrm{~T})$ is directed toward the north and inclined downward at $70.0^{\circ}$ to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earth's field.
-40 A wire 1.80 m long carries a current of 13.0 A and makes an angle of $35.0^{\circ}$ with a uniform magnetic field of magnitude $B=1.50 \mathrm{~T}$. Calculate the magnetic force on the wire.
$\bullet 41$ ILW A 13.0 g wire of length $L=62.0 \mathrm{~cm}$ is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (Fig. 28-41). What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads?


Figure 28-41 Problem 41.


Figure 28-42 Problem 42.
-42 The bent wire shown in Fig. 28-42 lies in a uniform magnetic field. Each straight section is 2.0 m long and makes an angle of $\theta=60^{\circ}$ with the $x$ axis, and the wire carries a current of 2.0 A . What is the net magnetic force on the wire in unitvector notation if the magnetic field is given by (a) 4.0 $\hat{\mathrm{k}} \mathrm{T}$ and (b) $4.0 \hat{\mathrm{i}} \mathrm{T}$ ? - 43 A single-turn current loop, carrying a current of 4.00 A , is in the shape of a right triangle with sides $50.0,120$, and 130 cm . The loop is in a uniform magnetic field of magnitude 75.0 mT whose direction is parallel to the current in the 130 cm side of the loop. What is the magnitude of the magnetic force on (a) the 130 cm side, (b) the 50.0 cm side, and (c) the 120 cm side? (d) What is the magnitude of the net force on the loop?

- 44 Figure 28-43 shows a wire ring of radius $a=1.8 \mathrm{~cm}$ that is perpendicular to the general direction of a radially symmetric, diverging magnetic field. The magnetic field at the ring is everywhere of the same magnitude $B=3.4 \mathrm{mT}$, and its direction at the ring everywhere


Figure 28-43 Problem 44. makes an angle $\theta=20^{\circ}$ with a normal to the plane of the ring. The twisted lead wires have no effect on the problem. Find the magnitude of the force the field exerts on the ring if the ring carries a current $i=4.6 \mathrm{~mA}$.
$\bullet 45$ A wire 50.0 cm long carries a 0.500 A current in the positive direction of an $x$ axis through a magnetic field $\vec{B}=$ $(3.00 \mathrm{mT}) \hat{\mathrm{j}}+(10.0 \mathrm{mT}) \hat{\mathrm{k}}$. In unit-vector notation, what is the magnetic force on the wire?
$\bullet$ •46 In Fig. 28-44, a metal wire of mass $m=24.1 \mathrm{mg}$ can slide with negligible friction on two horizontal parallel rails separated by distance $d=2.56 \mathrm{~cm}$. The track lies in a vertical uniform magnetic field of magnitude 56.3 mT . At time $t=0$, device $G$ is connected to the rails, producing a constant current $i=9.13 \mathrm{~mA}$ in the wire and rails (even as the wire moves). At $t=61.1 \mathrm{~ms}$, what are the wire's (a) speed and (b) direction of motion (left or right)?


Figure 28-44 Problem 46.
-๑.47 ©o A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60 . What are the (a) magnitude and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding?
$\bullet \bullet 48$ ©० A long, rigid conductor, lying along an $x$ axis, carries $\xrightarrow{\text { a current }}$ of 5.0 A in the negative $x$ direction. A magnetic field $\vec{B}$ is present, given by $\vec{B}=3.0 \hat{\mathrm{i}}+8.0 x^{2} \hat{\mathrm{j}}$, with $x$ in meters and $\vec{B}$ in milliteslas. Find, in unit-vector notation, the force on the 2.0 m seg ment of the conductor that lies between $x=1.0 \mathrm{~m}$ and $x=3.0 \mathrm{~m}$.

## Module 28-7 Torque on a Current Loop

- 49 SSM Figure $28-45$ shows a rectangular 20-turn coil of wire, of dimensions 10 cm by 5.0 cm . It carries a current of 0.10 A and is hinged along one long side. It is mounted in the $x y$ plane, at angle $\theta=30^{\circ}$ to the direction of a uniform magnetic field of magnitude 0.50 T . In unit-vector notation, what is the torque acting on the coil about the hinge line?
- 50 An electron moves in a cir-


Figure 28-45 Problem 49. cle of radius $r=5.29 \times 10^{-11} \mathrm{~m}$ with speed $2.19 \times 10^{6} \mathrm{~m} / \mathrm{s}$. Treat the circular path as a current loop with a constant current equal to the ratio of the electron's charge magnitude to the period of the motion. If the circle lies in a uniform magnetic field of magnitude $B=7.10 \mathrm{mT}$, what is the maximum possible magnitude of the torque produced on the loop by the field?

- 51 Figure 28-46 shows a wood cylinder of mass $m=0.250 \mathrm{~kg}$ and length $L=0.100 \mathrm{~m}$, with $N=10.0$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle $\theta$ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T , what is the least current $i$ through the coil that keeps the cylinder from rolling down the plane?


Figure 28-46 Problem 51.
-•52 In Fig. 28-47, a rectangular loop carrying current lies in the plane of a uniform magnetic field of magnitude 0.040 T . The loop consists of a single turn of flexible conducting wire that is wrapped around a flexible mount such that the dimensions of the rectangle can be changed. (The total length of the wire is not changed.) As edge length $x$ is varied from approximately zero


Figure 28-47
Problem 52. to its maximum value of approximately 4.0 cm , the magnitude $\tau$ of the torque on the loop changes. The maximum value of $\tau$ is $4.80 \times 10^{-8} \mathrm{~N} \cdot \mathrm{~m}$. What is the current in the loop?

- 53 Prove that the relation $\tau=N i A B \sin \theta$ holds not only for the rectangular loop of Fig. 28-19 but also for a closed loop of any shape. (Hint: Replace the loop of arbitrary shape with an assembly of adjacent long, thin, approximately rectangular loops that are nearly equivalent to the loop of arbitrary shape as far as the distribution of current is concerned.)


## Module 28-8 The Magnetic Dipole Moment

-54 A magnetic dipole with a dipole moment of magnitude $0.020 \mathrm{~J} / \mathrm{T}$ is released from rest in a uniform magnetic field of magnitude 52 mT . The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientation where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.80 mJ . (a) What is the initial angle between the dipole moment and the magnetic field? (b) What is the angle when the dipole is next (momentarily) at rest?
$\bullet 55$ SSM Two concentric, circular wire loops, of radii $r_{1}=20.0 \mathrm{~cm}$ and $r_{2}=30.0 \mathrm{~cm}$, are located in an $x y$ plane; each carries a clockwise current of 7.00 A (Fig. 28-48). (a) Find the magnitude of the net magnetic dipole moment of the system. (b) Repeat for reversed current in the inner loop.
-56 A circular wire loop of radius 15.0 cm carries a current of 2.60 A . It is placed so that the normal to its plane makes an angle of $41.0^{\circ}$ with


Figure 28-48 Problem 55. a uniform magnetic field of magnitude 12.0 T . (a) Calculate the magnitude of the magnetic dipole moment of the loop. (b) What is the magnitude of the torque acting on the loop?
-57 SSM A circular coil of 160 turns has a radius of 1.90 cm . (a) Calculate the current that results in a magnetic dipole moment of magnitude $2.30 \mathrm{~A} \cdot \mathrm{~m}^{2}$. (b) Find the maximum magnitude of the torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field.
-58 The magnetic dipole moment of Earth has magnitude $8.00 \times 10^{22} \mathrm{~J} / \mathrm{T}$. Assume that this is produced by charges flowing in Earth's molten outer core. If the radius of their circular path is 3500 km , calculate the current they produce.
-59 A current loop, carrying a current of 5.0 A , is in the shape of a right triangle with sides 30,40 , and 50 cm . The loop is in a uniform magnetic field of magnitude 80 mT whose direction is parallel to the current in the 50 cm side of the loop. Find the magnitude of (a) the magnetic dipole moment of the loop and (b) the torque on the loop.
-•60 Figure 28-49 shows a current loop $A B C D E F A$ carrying a current $i=5.00 \mathrm{~A}$. The sides of the loop are parallel to the coordinate axes shown, with $A B=20.0 \mathrm{~cm}, B C=30.0 \mathrm{~cm}$, and $F A=10.0 \mathrm{~cm}$. In unit-vector notation, what is the magnetic dipole moment of this loop? (Hint: Imagine equal and opposite currents $i$ in the line segment $A D$; then treat the two rectangular loops $A B C D A$ and $A D E F A$.)


Figure 28-49 Problem 60.
-•61 SSM The coil in Fig. 28-50 carries current $i=2.00 \mathrm{~A}$ in the direction indicated, is parallel to an $x z$ plane, has 3.00 turns and an area of $4.00 \times 10^{-3} \mathrm{~m}^{2}$, and lies in a uniform magnetic field $\vec{B}=(2.00 \hat{\mathrm{i}}-3.00 \hat{\mathrm{j}}-4.00 \hat{\mathrm{k}}) \mathrm{mT}$. What are (a) the orientation energy of the coil in the magnetic field and (b) the torque (in unit-vector notation) on the coil due to the magnetic field?


Figure 28-50 Problem 61.
-•62 60 In Fig. 28-51a, two concentric coils, lying in the same plane, carry currents in opposite directions. The current in the larger coil 1 is fixed. Current $i_{2}$ in coil 2 can be varied. Figure $28-51 b$ gives the net magnetic moment of the two-coil system as a function of $i_{2}$. The vertical axis scale is set by $\mu_{\mathrm{net}, \mathrm{s}}=2.0 \times 10^{-5} \mathrm{~A} \cdot \mathrm{~m}^{2}$, and the horizontal axis scale is set by $i_{2 s}=10.0 \mathrm{~mA}$. If the current in coil 2 is then reversed, what is the magnitude of the net magnetic moment of the two-coil system when $i_{2}=7.0 \mathrm{~mA}$ ?


Figure 28-51 Problem 62.

- 63 A circular loop of wire having a radius of 8.0 cm carries a current of 0.20 A . A vector of unit length and parallel to the dipole moment $\vec{\mu}$ of the loop is given by $0.60 \hat{\mathrm{i}}-0.80 \hat{\mathrm{j}}$. (This unit vector gives the orientation of the magnetic dipole moment vector.) If the loop is located in a uniform magnetic field given by $\vec{B}=$ $(0.25 \mathrm{~T}) \hat{\mathrm{i}}+(0.30 \mathrm{~T}) \hat{\mathrm{k}}$, find (a) the torque on the loop (in unitvector notation) and (b) the orientation energy of the loop.
- 64 ©o Figure $28-52$ gives the orientation energy $U$ of a magnetic dipole in an external magnetic field $\vec{B}$, as a function of angle $\phi$ between the directions of $\vec{B}$ and the dipole moment. The vertical axis scale is set by $U_{s}=2.0 \times 10^{-4} \mathrm{~J}$. The dipole can be rotated about an axle with negligible friction in order to change $\phi$. Counterclockwise rotation from $\phi=0$ yields positive values of $\phi$, and clockwise
rotations yield negative values. The dipole is to be released at angle $\phi=0$ with a rotational kinetic energy of $6.7 \times 10^{-4} \mathrm{~J}$, so that it rotates counterclockwise. To what maximum value of $\phi$ will it rotate? (In the language of Module 8-3, what value $\phi$ is the turning point in the potential well of Fig. 28-52?)


Figure 28-52 Problem 64.
-•65 SSM ILW A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field $\vec{B}$ of magnitude 5.71 mT . If the torque on the coil from the field is maximized, what are (a) the angle between $\vec{B}$ and the coil's magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?

## Additional Problems

66 A proton of charge $+e$ and mass $m$ enters a uniform magnetic field $\vec{B}=B \hat{\mathrm{i}}$ with an initial velocity $\vec{v}=v_{0 x} \hat{\mathrm{i}}+v_{0, y} \hat{\mathrm{j}}$. Find an expression in unit-vector notation for its velocity $\vec{v}$ at any later time $t$.
67 A stationary circular wall clock has a face with a radius of 15 cm . Six turns of wire are wound around its perimeter; the wire carries a current of 2.0 A in the clockwise direction. The clock is located where there is a constant, uniform external magnetic field of magnitude 70 mT (but the clock still keeps perfect time). At exactly 1:00 P.M., the hour hand of the clock points in the direction of the external magnetic field. (a) After how many minutes will the minute hand point in the direction of the torque on the winding due to the magnetic field? (b) Find the torque magnitude.
68 A wire lying along a $y$ axis from $y=0$ to $y=0.250 \mathrm{~m}$ carries a current of 2.00 mA in the negative direction of the axis. $\vec{B}$ The wire fully lies in a nonuniform magnetic field that is given by $\vec{B}=(0.300 \mathrm{~T} / \mathrm{m}) y \hat{\mathrm{i}}+(0.400 \mathrm{~T} / \mathrm{m}) y \hat{\mathrm{j}}$. In unit-vector notation, what is the magnetic force on the wire?
69 Atom 1 of mass $35 u$ and atom 2 of mass $37 u$ are both singly ionized with a charge of $+e$. After being introduced into a mass spectrometer (Fig. 28-12) and accelerated from rest through a potential difference $V=7.3 \mathrm{kV}$, each ion follows a circular path in a uniform magnetic field of magnitude $B=0.50 \mathrm{~T}$. What is the distance $\Delta x$ between the points where the ions strike the detector?

70 An electron with kinetic energy 2.5 keV moving along the positive direction of an $x$ axis enters a region in which a uniform electric field of magnitude $10 \mathrm{kV} / \mathrm{m}$ is in the negative direction of the $y$ axis. A uniform magnetic field $\vec{B}$ is to be set up to keep the electron moving along the $x$ axis, and the direction of $\vec{B}$ is to be chosen to minimize the required magnitude of $\vec{B}$. In unit-vector notation, what $\vec{B}$ should be set up?
71 Physicist S. A. Goudsmit devised a method for measuring the mass of heavy ions by timing their period of revolution in a known magnetic field. A singly charged ion of iodine makes 7.00 rev in a 45.0 mT field in 1.29 ms . Calculate its mass in atomic mass units.

72 A beam of electrons whose kinetic energy is $K$ emerges from a thin-foil "window" at the end of an accelerator tube. A metal plate at distance $d$ from this window is perpendicular to the direction of the emerging beam (Fig. 28-53). (a) Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field such that

$$
B \geq \sqrt{\frac{2 m K}{e^{2} d^{2}}},
$$

in which $m$ and $e$ are the electron mass and charge. (b) How should $\vec{B}$ be oriented?
73 ssm At time $t=0$, an electron with kinetic energy 12 keV moves through $x=0$ in the positive direction of an $x$ axis that is parallel to the horizontal component of Earth's magnetic field $\vec{B}$. The field's vertical component is downward and has magnitude $55.0 \mu \mathrm{~T}$. (a) What is the magnitude of the electron's acceleration due to $\vec{B}$ ? (b) What is the electron's distance from the $x$ axis when the electron reaches coordinate $x=20 \mathrm{~cm}$ ?
74 © A particle with charge 2.0 C moves through a uniform magnetic field. At one instant the velocity of the particle is $(2.0 \hat{\mathrm{i}}+4.0 \hat{\mathrm{j}}+6.0 \hat{\mathrm{k}}) \mathrm{m} / \mathrm{s}$ and the magnetic force on the particle is $(4.0 \hat{\mathrm{i}}-20 \hat{\mathrm{j}}+12 \hat{\mathrm{k}}) \xrightarrow{\mathrm{N}}$. The $x$ and $y$ components of the magnetic field are equal. What is $\vec{B}$ ?

75 A proton, a deuteron ( $q=+e, m=2.0 \mathrm{u}$ ), and an alpha particle $(q=+2 e, m=4.0 \mathrm{u})$ all having the same kinetic energy enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. What is the ratio of (a) the radius $r_{d}$ of the deuteron path to the radius $r_{p}$ of the proton path and (b) the radius $r_{\alpha}$ of the alpha particle path to $r_{p}$ ?
76 Bainbridge's mass spectrometer, shown in Fig. 28-54, separates ions having the same velocity. The ions, after entering through slits, $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$, pass through a velocity selector composed of an electric field produced by the charged plates P and $\mathrm{P}^{\prime}$, and a magnetic field $\vec{B}$ perpendicular to the electric field and the ion path. The ions that then pass undeviated through the crossed $\vec{E}$ and $\vec{B}$ fields


Figure 28-54 Problem 76. enter into a region where a second magnetic field $\vec{B}^{\prime}$ exists, where they are made to follow circular paths. A photographic plate (or a modern detector) registers their arrival. Show that, for the ions, $q / m=E / r B B^{\prime}$, where $r$ is the radius of the circular orbit.

77 ssm In Fig. 28-55, an electron moves at speed $v=100 \mathrm{~m} / \mathrm{s}$ along an $x$ axis through uniform electric and magnetic fields. The magnetic field $\vec{B}$ is directed into the page and


Figure 28-55 Problem 77. has magnitude 5.00 T. In unit-vector notation, what is the electric field?
78 (a) In Fig. 28-8, show that the ratio of the Hall electric field magnitude $E$ to the magnitude $E_{C}$ of the electric field responsible for moving charge (the current) along the length of
the strip is

$$
\frac{E}{E_{C}}=\frac{B}{n e \rho},
$$

where $\rho$ is the resistivity of the material and $n$ is the number density of the charge carriers. (b) Compute this ratio numerically for Problem 13. (See Table 26-1.)
79 SSM A proton, a deuteron ( $q=+e, m=2.0 \mathrm{u}$ ), and an alpha particle ( $q=+2 e, m=4.0 \mathrm{u}$ ) are accelerated through the same potential difference and then enter the same region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. What is the ratio of (a) the proton's kinetic energy $K_{p}$ to the alpha particle's kinetic energy $K_{\alpha}$ and (b) the deuteron's kinetic energy $K_{d}$ to $K_{\alpha}$ ? If the radius of the proton's circular path is 10 cm , what is the radius of (c) the deuteron's path and (d) the alpha particle's path?

80 An electron is moving at $7.20 \times 10^{6} \mathrm{~m} / \mathrm{s}$ in a magnetic field of strength 83.0 mT . What is the (a) maximum and (b) minimum magnitude of the force acting on the electron due to the field? (c) At one point the electron has an acceleration of magnitude $4.90 \times 10^{14} \mathrm{~m} / \mathrm{s}^{2}$. What is the angle between the electron's velocity and the magnetic field?
81 A $5.0 \mu \mathrm{C}$ particle moves through a region containing the uniform magnetic field $-20 \hat{\mathrm{i}} \mathrm{mT}$ and the uniform electric field $300 \hat{\mathrm{j}} \mathrm{V} / \mathrm{m}$. At a certain instant the velocity of the particle is $(17 \hat{i}-11 \hat{j}+7.0 \hat{k}) \mathrm{km} / \mathrm{s}$. At that instant and in unit-vector notation, what is the net electromagnetic force (the sum of the electric and magnetic forces) on the particle?

82 In a Hall-effect experiment, a current of 3.0 A sent lengthwise through a conductor 1.0 cm wide, 4.0 cm long, and $10 \mu \mathrm{~m}$ thick produces a transverse (across the width) Hall potential difference of $10 \mu \mathrm{~V}$ when a magnetic field of 1.5 T is passed perpendicularly through the thickness of the conductor. From these data, find (a) the drift velocity of the charge carriers and (b) the number density of charge carriers. (c) Show on a diagram the polarity of the Hall potential difference with assumed current and magnetic field directions, assuming also that the charge carriers are electrons.
83 SSM A particle of mass 6.0 g moves at $4.0 \mathrm{~km} / \mathrm{s}$ in an $x y$ plane, in a region with a uniform magnetic field given by $5.0 \hat{\mathrm{i}} \mathrm{mT}$. At one instant, when the particle's velocity is directed $37^{\circ}$ counterclockwise from the positive direction of the $x$ axis, the magnetic force on the particle is $0.48 \hat{\mathrm{k}} \mathrm{N}$. What is the particle's charge?
84 A wire lying along an $x$ axis from $x=0$ to $x=1.00 \mathrm{~m}$ carries a current of 3.00 A in the positive $x$ direction. The wire is immersed in a nonuniform magnetic field that is given by $\vec{B}=$ $\left(4.00 \mathrm{~T} / \mathrm{m}^{2}\right) x^{2} \hat{\mathrm{i}}-\left(0.600 \mathrm{~T} / \mathrm{m}^{2}\right) x^{2} \hat{\mathrm{j}}$. In unit-vector notation, what is the magnetic force on the wire?
85 At one instant, $\vec{v}=(-2.00 \hat{i}+4.00 \hat{\mathrm{j}}-6.00 \hat{\mathrm{k}}) \mathrm{m} / \mathrm{s}$ is the velocity of a proton in a uniform magnetic field $\vec{B}=(2.00 \hat{\mathrm{i}}-$ $4.00 \hat{j}+8.00 \hat{k}) T$. At that instant, what are (a) the magnetic force $\vec{F}$ acting on the proton, in unit-vector notation, (b) the angle between $\vec{v}$ and $\vec{F}$, and (c) the angle between $\vec{v}$ and $\vec{B}$ ?
86 An electron has velocity $\vec{v}=(32 \hat{i}+40 \hat{j}) \mathrm{km} / \mathrm{s}$ as it enters a uniform magnetic field $\vec{B}=60 \hat{\hat{i}} \mu \mathrm{~T}$. What are (a) the radius of the helical path taken by the electron and (b) the pitch of that path? (c) To an observer looking into the magnetic field region from the entrance point of the electron, does the electron spiral clockwise or counterclockwise as it moves?

87 Figure 28-56 shows a homopolar generator, which has a solid conducting disk as rotor and which is rotated by a motor (not shown). Conducting brushes connect this emf device to a circuit through which the device drives current. The device can produce a greater emf than wire loop rotors because they can spin at a much higher angular speed without rupturing. The disk has radius $R=0.250 \mathrm{~m}$ and rotation frequency $f=4000 \mathrm{~Hz}$, and the device is in a uniform magnetic field of magnitude $B=60.0 \mathrm{mT}$ that is perpendicular to the disk. As the disk is rotated, conduction electrons along the conducting path (dashed line) are forced to move through the magnetic field. (a) For the indicated rotation, is the magnetic force on those electrons up or down in the figure? (b) Is the magnitude of that force greater at the rim or near the center of the disk? (c) What is the work per unit charge done in moving charge along the radial line, between the rim and the center? (d) What, then, is the emf of the device? (e) If the current is 50.0 A , what is the power at which electrical energy is being produced?


88 In Fig. 28-57, the two ends of a U-shaped wire of mass $m=$ 10.0 g and length $L=20.0 \mathrm{~cm}$ are immersed in mercury (which is a conductor). The wire is in a uniform field of magnitude $B=0.100 \mathrm{~T}$. A switch (unshown) is rapidly closed and then reopened, sending a pulse of current through the wire, which causes the wire to jump upward. If jump height $h=3.00 \mathrm{~m}$, how much charge was in the pulse? Assume that the duration of the pulse is much less than the time of flight. Consider the definition of impulse (Eq. 9-30)
and its relationship with momentum (Eq. 9-31). Also consider the relationship between charge and current (Eq. 26-2).


Figure 28-57 Problem 88.

89 In Fig. 28-58, an electron of mass $m$, charge $-e$, and low (negligible) speed enters the region between two plates of potential difference $V$ and plate separation $d$, initially headed directly toward the top plate. A uniform magnetic field of


Figure 28-58 Problem 89. magnitude $B$ is normal to the plane of the figure. Find the minimum value of $B$ such that the electron will not strike the top plate.

90 A particle of charge $q$ moves in a circle of radius $r$ with speed $v$. Treating the circular path as a current loop with an average current, find the maximum torque exerted on the loop by a uniform field of magnitude $B$.

91 In a Hall-effect experiment, express the number density of charge carriers in terms of the Hall-effect electric field magnitude $E$, the current density magnitude $J$, and the magnetic field magnitude $B$.

92 An electron that is moving through a uniform magnetic field has velocity $\vec{v}=(40 \mathrm{~km} / \mathrm{s}) \hat{\mathrm{i}}+(35 \mathrm{~km} / \mathrm{s}) \hat{\mathrm{j}}$ when it experiences a force $\vec{F}=-(4.2 \mathrm{fN}) \hat{\mathrm{i}}+(4.8 \mathrm{fN}) \hat{\mathrm{j}}$ due to the magnetic field. If $B_{x}=0$, calculate the magnetic field $\vec{B}$.

