9 A into the page, 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\oint \vec{B} \cdot d \vec{s}$ around each, greatest first.
10 Figure 29-33 shows four identical currents $i$ and five Amperian paths ( $a$ through $e$ ) encircling them. Rank the paths according to the value of $\oint \vec{B} \cdot d \vec{s}$ taken in the directions shown, most positive first.

11 Figure 29-34 shows three arrangements of three long straight wires carrying equal currents directly into or out of the page. (a) Rank the arrangements according to the magnitude of the net force on wire $A$ due to the currents in the other wires, greatest first. (b) In arrangement 3 , is the angle between the net force on wire $A$ and the dashed line equal to, less than, or more than $45^{\circ}$ ?


Figure 29-33 Question 10.


Figure 29-34 Question 11.

## Problems

## - - .. Number of dots indicates level of problem difficulty <br> Module 29-1 Magnetic Field Due to a Current

Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign
Worked-out solution available in Student Solutions Manual WWW Worked-out solution is at ILW Interactive solution is at Additional information available in The Flying Circus of Physics and at flyingcircusofphysics.com
-4 A straight conductor carrying
-1 A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A . (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is $20 \mu \mathrm{~T}$.
-2 Figure 29-35a shows an element of length $d s=1.00 \mu \mathrm{~m}$ in a very long straight wire carrying current. The current in that element sets up ${ }_{\vec{B}}$ a differential magnetic field $d \vec{B}$ at points in the surrounding space. Figure 29-35b

(a)

(b)

Figure 29-35 Problem 2. gives the magnitude $d B$ of the field for points 2.5 cm from the element, as a function of angle $\theta$ between the wire and a straight line to the point. The vertical scale is set by $d B_{s}=60.0 \mathrm{pT}$. What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm from the wire? -3 SSM At a certain location in the Philippines, Earth's magnetic field of $39 \mu \mathrm{~T}$ is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?
current $i=5.0 \mathrm{~A}$ splits into identical semicircular arcs as shown in Fig. 29-36. What is the magnetic field at the center $C$ of the resulting circular loop?
-5 In Fig. 29-37, a current $i=10 \mathrm{~A}$ is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R=5.0 \mathrm{~mm}$. Point $b$ is midway between the straight sections and so distant from the semicircle that each straight sec-


Figure 29-36 Problem 4.


Figure 29-37 Problem 5. tion can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of $\vec{B}$ at $a$ and the (c) magnitude and (d) direction of $\vec{B}$ at $b$ ?
$\bullet 6$ In Fig. 29-38, point $P$ is at perpendicular distance $R=2.00 \mathrm{~cm}$ from a very long straight wire carrying a current. The magnetic field $\vec{B}$ set up at point $P$ is due to contributions from all the identical current-length elements $i d \vec{s}$ along the wire. What is the distance $s$


Figure 29-38 Problem 6.
to the element making (a) the greatest contribution to field $\vec{B}$ and (b) $10.0 \%$ of the greatest contribution?
-7 60 In Fig. 29-39, two circular arcs have radii $a=13.5 \mathrm{~cm}$ and $b=10.7 \mathrm{~cm}$, subtend angle $\theta=74.0^{\circ}$, carry current $i=0.411 \mathrm{~A}$, and share the same center of curvature $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $P$ ?

- 8 In Fig. 29-40, two semicircular arcs have radii $R_{2}=7.80 \mathrm{~cm}$ and $R_{1}=$ 3.15 cm , carry current $i=0.281 \mathrm{~A}$, and have the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $C$ ?
$\bullet 9$ Ssm Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude $300 \mu \mathrm{~T}$. (a) Should the currents be in the same or opposite directions? (b) How much current is needed?
-10 In Fig. 29-41, a wire forms a semicircle of radius $R=9.26 \mathrm{~cm}$ and two (radial) straight segments each of length $L=13.1 \mathrm{~cm}$. The wire carries current $i=34.8 \mathrm{~mA}$. What are the


Figure 29-41 Problem 10. (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature $C$ ?
-11 In Fig. 29-42, two long straight wires are perpendicular to the page and separated by distance $d_{1}=0.75 \mathrm{~cm}$. Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point $P$ located at distance $d_{2}=1.50 \mathrm{~cm}$ from wire 2?
-12 In Fig. 29-43, two long straight wires at separation $d=16.0 \mathrm{~cm}$ carry currents $i_{1}=3.61 \mathrm{~mA}$ and $i_{2}=3.00 i_{1}$ out of the page. (a) Where on the $x$ axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1 , shifted toward wire 2, or unchanged?
$\bullet \bullet 13$ In Fig. 29-44, point $P_{1}$ is at distance $R=13.1 \mathrm{~cm}$ on the perpendicular bisector of a straight wire of length $L=18.0 \mathrm{~cm}$ carrying

Figure 29-44 Problems 13 and 17.

current $i=58.2 \mathrm{~mA}$. (Note that the wire is not long.) What is the magnitude of the magnetic field at $P_{1}$ due to $i$ ?
$\bullet 14$ Equation 29-4 gives the magnitude $B$ of the magnetic field set up by a current in an infinitely long straight wire, at a point $P$ at perpendicular distance $R$ from the wire. Suppose that point $P$ is actually at perpendicular distance $R$ from the midpoint of a wire with a finite length $L$. Using Eq. 29-4 to calculate $B$ then results in a certain percentage error. What value must the ratio $L / R$ exceed if the percentage error is to be less than $1.00 \%$ ? That is, what $L / R$ gives

$$
\frac{(B \text { from Eq. } 29-4)-(B \text { actual })}{(B \text { actual })}(100 \%)=1.00 \% ?
$$

- 15 Figure 29-45 shows two current segments. The lower segment carries a current of $i_{1}=0.40 \mathrm{~A}$ and includes a semicircular arc with radius 5.0 cm , angle $180^{\circ}$, and center point $P$. The upper segment carries current $i_{2}=2 i_{1}$ and includes a circular arc with radius 4.0 cm , angle


Figure 29-45 Problem 15. $120^{\circ}$, and the same center point $P$. What are the (a) magnitude and (b) direction of the net magnetic field $\vec{B}$ at $P$ for the indicated current directions? What are the (c) magnitude and (d) direction of $\vec{B}$ if $i_{1}$ is reversed?
-•16 60 In Fig. 29-46, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has


Figure 29-46 Problem 16. radius 1.50 cm and carries 4.00 mA . Loop 2 has radius 2.50 cm and carries 6.00 mA . Loop 2 is to be rotated about a diameter while the net magnetic field $\vec{B}$ set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT ?
$\bullet 17$ SSM In Fig. 29-44, point $P_{2}$ is at perpendicular distance $R=25.1 \mathrm{~cm}$ from one end of a straight wire of length $L=13.6 \mathrm{~cm}$ carrying current $i=0.693 \mathrm{~A}$. (Note that the wire is not long.) What is the magnitude of the magnetic field at $P_{2}$ ?
-•18 A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm , a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-47a shows the arrange-

(a)

Figure 29-47 Problem 18. ment but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is $47.25 \mu \mathrm{~T}$. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-47b). The magnetic field produced at the (same) center of curvature now has magnitude $15.75 \mu \mathrm{~T}$, and its direction is reversed from the initial magnetic field. What is the radius of the smaller semicircle?

- 19 One long wire lies along an $x$ axis and carries a current of 30 A in the positive $x$ direction. A second long wire is perpendicular to the $x y$ plane, passes through the point ( $0,4.0 \mathrm{~m}, 0$ ), and carries a current of 40 A in the positive $z$ direction. What is the magnitude of the resulting magnetic field at the point $(0,2.0 \mathrm{~m}, 0)$ ?
-20 In Fig. 29-48, part of a long insulated wire carrying current $i=5.78 \mathrm{~mA}$ is bent into a circular section of radius $R=1.89 \mathrm{~cm}$. In unit-vector notation, what is the magnetic field at the center of curvature $C$ if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated $90^{\circ}$ counterclockwise as indicated?
- 21

Figure 29-49 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance $d_{1}=6.00 \mathrm{~m}$ and distance $d_{2}=4.00 \mathrm{~m}$. What is the magnitude of the net magnetic field at point $P$, which lies on a perpendicular bisector to the wires?

- 22 (6) Figure 29-50a shows, in cross section, two long, parallel wires carrying current and separated by distance $L$. The ratio $i_{1} / i_{2}$ of their currents is 4.00; the directions of the currents are not indicated. Figure 29-50b shows the $y$ component $B_{y}$ of their net magnetic field along the $x$ axis to the right of wire 2 . The vertical scale is set by $B_{y s}=4.0 \mathrm{nT}$, and the horizontal scale is set by $x_{s}=20.0 \mathrm{~cm}$.
(a) At what value of $x>0$ is $B_{y}$ maximum? (b) If $i_{2}=3 \mathrm{~mA}$, what is the value of that maximum? What is the direction (into or out of the page) of (c) $i_{1}$ and (d) $i_{2}$ ?


Figure 29-50 Problem 22.

- 23 ILW Figure $29-51$ shows a snapshot of a proton moving at velocity $\vec{v}=(-200 \mathrm{~m} / \mathrm{s}) \hat{\mathrm{j}}$ toward a long straight wire with current $i=350 \mathrm{~mA}$. At the instant shown, the proton's distance from the wire is $d=2.89 \mathrm{~cm}$. In unit-vector notation, what is the magnetic force on the proton due to the current?
- 24 so Figure 29-52 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance $d=15.0 \mathrm{~cm}$ from the origin of the coordinate system, where they create a net magnetic field $\vec{B}$. (a) To what value of $x$ must you move wire 1 along the $x$ axis in order to rotate $\vec{B}$ counterclockwise


Figure 29-51 Problem 23.


Figure 29-52
Problem 24.
by $30^{\circ}$ ? (b) With wire 1 in that new position, to what value of $x$ must you move wire 3 along the $x$ axis to rotate $\vec{B}$ by $30^{\circ}$ back to its initial orientation?

- 25 SSM A wire with current $i=3.00 \mathrm{~A}$ is shown in Fig. 29-53. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle $\theta$ and runs along the circumference of the circle. The arc and


Figure 29-53
Problem 25. the two straight sections all lie in the same plane. If $B=0$ at the circle's center, what is $\theta$ ? - 26 (50 In Fig. 29-54a, wire 1 consists of a circular arc and two radial lengths; it carries current $i_{1}=0.50 \mathrm{~A}$ in the direction indicated. Wire 2 , shown in cross section, is long, straight, and perpendicular to the plane of the figure. Its distance from the center of the arc is equal to the radius $R$ of the arc, and it carries a current $i_{2}$ that can be varied. The two currents set up a net magnetic field $\vec{B}$ at the center of the arc. Figure $29-54 b$ gives the square of the field's magnitude $B^{2}$ plotted versus the square of the current $i_{2}^{2}$. The vertical scale is set by $B_{s}^{2}=10.0 \times 10^{-10} \mathrm{~T}^{2}$. What angle is subtended by the arc?


Figure 29-54 Problem 26.
$\bullet 27$ In Fig. 29-55, two long straight wires (shown in cross section) carry the currents $i_{1}=30.0 \mathrm{~mA}$ and $i_{2}=$ 40.0 mA directly out of the page. They are equal distances from the origin, where they set up a magnetic field $\vec{B}$. To what value must current $i_{1}$ be changed in order to rotate $\vec{B} 20.0^{\circ}$ clockwise?


Figure 29-55 Problem 27.
$\bullet 28$ so Figure 29-56a shows two wires, each carrying a current. Wire 1 consists of a circular arc of radius $R$ and two radial lengths;


Figure 29-56 Problem 28.
it carries current $i_{1}=2.0 \mathrm{~A}$ in the direction indicated. Wire 2 is long and straight; it carries a current $i_{2}$ that can be varied; and it is $\xrightarrow{\text { at distance } R / 2 \text { from the center of the arc. The net magnetic field }}$ $\vec{B}$ due to the two currents is measured at the center of curvature of the arc. Figure $29-56 b$ is a plot of the component of $\vec{B}$ in the direction perpendicular to the figure as a function of current $i_{2}$. The horizontal scale is set by $i_{2 s}=1.00 \mathrm{~A}$. What is the angle subtended by the arc?
-•29 SSM In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a=20 \mathrm{~cm}$. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A . In unitvector notation, what is the net magnetic field at the square's center?

Figure 29-57 Problems 29, 37, and 40.

-•30 Two long straight thin wires with current lie against an equally long plastic cylinder, at radius $R=20.0 \mathrm{~cm}$ from the cylinder's central axis. Figure $29-58 a$ shows, in cross section, the cylinder and wire 1 but not wire 2 . With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle $\theta_{1}=0^{\circ}$ to angle $\theta_{1}=180^{\circ}$, through the first and second quadrants of the $x y$ coordinate system. The net magnetic field $\vec{B}$ at the center of the cylinder is measured as a function of $\theta_{1}$. Figure 29-58b gives the $x$ component $B_{x}$ of that field as a function of $\theta_{1}$ (the vertical scale is set by $B_{x s}=6.0 \mu \mathrm{~T}$ ), and Fig. 29-58c gives the $y$ component $B_{y}$ (the vertical scale is set by $B_{y s}=$ $4.0 \mu \mathrm{~T}$ ). (a) At what angle $\theta_{2}$ is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current in wire 1 and the (d) size and (e) direction of the current in wire 2 ?
(a)

(c)


(b)

Figure 29-58 Problem 30.
$\bullet \bullet 31$ In Fig. 29-59, length $a$ is 4.7 cm (short) and current $i$ is 13 A . What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at point $P$ ?
-••32

- The current-carrying wire loop in Fig. 29-60a lies all in one plane and consists of a semicircle of radius 10.0 cm , a smaller semicircle with the same center, and two radial lengths. The smaller semicircle is rotated out of that plane by angle $\theta$, until it is perpendicular to the plane (Fig. 29-60b). Figure 29-60c gives the magnitude of the net magnetic field at the center of curvature versus angle $\theta$. The vertical scale is set by $B_{a}=10.0 \mu \mathrm{~T}$ and $B_{b}=12.0 \mu \mathrm{~T}$. What is the radius of the smaller semicircle?


(c)

Figure 29-60 Problem 32.
-••33 SSM ILW Figure 29-61 shows a cross section of a long thin ribbon of width $w=4.91 \mathrm{~cm}$ that is carrying a uniformly distributed total current $i=4.61 \mu \mathrm{~A}$ into the page. In unitvector notation, what is the magnetic field $\vec{B}$ at a point $P$ in the plane of the ribbon at a distance $d=2.16 \mathrm{~cm}$ from its edge? (Hint: Imagine the ribbon as being constructed from many long, thin, parallel wires.)
-••34 ©0 Figure 29-62 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm . Wire 1 carries current $i_{1}=60.0 \mathrm{~mA}$ out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current $i_{2}=40.0 \mathrm{~mA}$ out of the page and can be moved around the cylinder. At what (positive) angle $\theta_{2}$ should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude 80.0 nT ?

## Module 29-2 Force Between Two Parallel Currents

-35 SSM Figure 29-63 shows wire 1 in cross section; the wire is long


Figure 29-59 Problem 31.
and straight, carries a current of 4.00 mA out of the page, and is at distance $d_{1}=2.40 \mathrm{~cm}$ from a surface. Wire 2 , which is parallel to wire 1 and also long, is at horizontal distance $d_{2}=5.00 \mathrm{~cm}$ from wire 1 and carries a current of 6.80 mA into the page. What is the $x$ component of the magnetic force per unit length on wire 2 due to wire 1 ?

- 36 In Fig. 29-64, five long parallel wires in an $x y$ plane are separated by distance $d=8.00 \mathrm{~cm}$, have lengths of 10.0 m , and carry identical currents of 3.00 A out of the page. Each wire experiences a magnetic force due


Figure 29-64 Problems 36 and 39. to the currents in the other wires. In unit-vector notation, what is the net magnetic force on (a) wire 1 , (b) wire 2 , (c) wire 3 , (d) wire 4 , and (e) wire 5?
-•37 60 In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a=13.5 \mathrm{~cm}$. Each wire carries 7.50 A , and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unitvector notation, what is the net magnetic force per meter of wire length on wire 4 ?
$\bullet$ • 38 Fo Figure 29-65a shows, in cross section, three currentcarrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an $x$ axis, with separation $d$. Wire 1 has a current of 0.750 A , but the direction of the current is not given. Wire 3 , with a current of 0.250 A out of the page, can be moved along the $x$ axis to the right of wire 2 . As wire 3 is moved, the magnitude of the net magnetic force $\vec{F}_{2}$ on wire 2 due to the currents in wires 1 and 3 changes. The $x$ component of that force is $F_{2 x}$ and the value per unit length of wire 2 is $F_{2 x} / L_{2}$. Figure 29-65b gives $F_{2 x} / L_{2}$ versus the position $x$ of wire 3 . The plot has an asymptote $F_{2 x} / L_{2}=-0.627 \mu \mathrm{~N} / \mathrm{m}$ as $x \rightarrow \infty$. The horizontal scale is set by $x_{s}=12.0 \mathrm{~cm}$. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2 ?


Figure 29-65 Problem 38.
-•39 (60 In Fig. 29-64, five long parallel wires in an $x y$ plane are separated by distance $d=50.0 \mathrm{~cm}$. The currents into the page are $i_{1}=2.00 \mathrm{~A}, i_{3}=0.250 \mathrm{~A}, i_{4}=4.00 \mathrm{~A}$, and $i_{5}=2.00 \mathrm{~A}$; the current out of the page is $i_{2}=4.00 \mathrm{~A}$. What is the magnitude of the net force per unit length acting on wire 3 due to the currents in the other wires?

- 40 In Fig. 29-57, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a=8.50 \mathrm{~cm}$. Each wire carries 15.0 A , and all the currents are out of the page. In unit-vector notation, what is the net magnetic force per meter of wire length on wire 1 ?
-••41 ILW In Fig. 29-66, a long straight wire carries a current $i_{1}=30.0 \mathrm{~A}$ and a rectangular loop carries current $i_{2}=20.0 \mathrm{~A}$. Take the dimensions to be $a=1.00 \mathrm{~cm}$, $b=8.00 \mathrm{~cm}$, and $L=30.0 \mathrm{~cm}$. In unit-vector notation, what is the net force on the loop due to $i_{1}$ ?


## Module 29-3 Ampere's Law

-42 In a particular region there is a uniform current density of


Figure 29-66 Problem 41. $15 \mathrm{~A} / \mathrm{m}^{2}$ in the positive $z$ direction. What is the value of $\oint \vec{B} \cdot d \vec{s}$ when that line integral is calculated along a closed path consisting of the three straight-line segments from $(x, y, z)$ coordinates $(4 d, 0,0)$ to $(4 d, 3 d, 0)$ to $(0,0,0)$ to $(4 d, 0,0)$, where $d=20 \mathrm{~cm}$ ?

- 43 Figure 29-67 shows a cross section across a diameter of a long cylindrical conductor of radius $a=2.00 \mathrm{~cm}$ carrying uniform current 170 A . What is the magnitude of the current's magnetic field at radial distance (a) 0 , (b) 1.00 cm , (c) 2.00 cm (wire's surface),


Figure 29-67
Problem 43. and (d) 4.00 cm ?
-44 Figure 29-68 shows two closed paths wrapped around two conducting loops carrying currents $i_{1}=5.0 \mathrm{~A}$ and $i_{2}=3.0 \mathrm{~A}$. What is the value of the integral $\oint \vec{B} \cdot d \vec{s}$ for (a) path 1 and (b) path 2?


Figure 29-68 Problem 44.
-45 SSM Each of the eight conductors in Fig. 29-69 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d \vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2 ?


Figure 29-69 Problem 45.
-46 Eight wires cut the page perpendicularly at the points shown in Fig. 29-70. A wire labeled with the integer $k(k=1,2, \ldots, 8)$ carries the current $k i$, where $i=4.50 \mathrm{~mA}$. For those wires with odd $k$, the current is out of the page; for those with even $k$, it is into the page. Evaluate $\oint \vec{B} \cdot d \vec{s}$ along the closed path indicated and in the direction shown.

- 47 ILW The current density $\vec{J}$


Figure 29-70 Problem 46. inside a long, solid, cylindrical wire of radius $a=3.1 \mathrm{~mm}$ is in the direction of the central axis, and its magnitude varies linearly with radial distance $r$ from the axis
according to $J=J_{0} r / a$, where $J_{0}=310 \mathrm{~A} / \mathrm{m}^{2}$. Find the magnitude of the magnetic field at (a) $r=0$, (b) $r=a / 2$, and (c) $r=a$.

- 48 In Fig. 29-71, a long circular pipe with outside radius $R=2.6 \mathrm{~cm}$ carries a (uniformly distributed) current $i=8.00 \mathrm{~mA}$ into the page. A wire runs parallel to the pipe at a distance of 3.00 R from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point $P$ has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.


Figure 29-71
Problem 48.

## Module 29-4 Solenoids and Toroids

-49 A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A . (It is made up of a square solenoid - instead of a round one as in Fig. 29-17 - bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?
-50 A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1200 turns; it carries a current of 3.60 A . Calculate the magnitude of the magnetic field inside the solenoid.
-51 A 200-turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A . Calculate the magnitude of the magnetic field $\vec{B}$ inside the solenoid.
-52 A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A . The magnetic field inside the solenoid is 23.0 mT . Find the length of the wire forming the solenoid.

- 53 A long solenoid has 100 turns/cm and carries current $i$. An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is $0.0460 c(c=$ speed of light). Find the current $i$ in the solenoid.
$\bullet \bullet 54$ An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is $800 \mathrm{~m} / \mathrm{s}$ and its velocity vector makes an angle of $30^{\circ}$ with the central axis of the solenoid. The solenoid carries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)
$\bullet \bullet 55$ SSM ILW www A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA . A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at $45.0^{\circ}$ to the axial direction? (b) What is the magnitude of the magnetic field there?


## Module 29-5 A Current-Carrying Coil as a Magnetic Dipole

-56 Figure 29-72 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius $R=25.0 \mathrm{~cm}$, separated by a distance


Figure 29-72
Problem 56.
$s=R$. The two coils carry equal currents $i=12.2 \mathrm{~mA}$ in the same direction. Find the magnitude of the net magnetic field at $P$, midway between the coils.

- 57 SSM A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter $d=5.0 \mathrm{~cm}$. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude $5.0 \mu \mathrm{~T}$ (approximately one-tenth that of Earth's magnetic field)?
-58 Figure 29-73a shows a length of wire carrying a current $i$ and bent into a circular coil of one turn. In Fig. 29-73b the same length of wire has been bent to give a coil of two turns, each of half the original radius. (a) If $B_{a}$ and $B_{b}$ are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio $B_{b} / B_{a}$ ? (b) What is the ratio $\mu_{b} / \mu_{a}$ of the dipole moment magnitudes

(a)

Figure 29-73 Problem 58. of the coils?
-59 SSM What is the magnitude of the magnetic dipole moment $\vec{\mu}$ of the solenoid described in Problem 51?

- 60 ( 60 Fig. 29-74a, two circular loops, with different currents but the same radius of 4.0 cm , are centered on a $y$ axis. They are initially separated by distance $L=3.0 \mathrm{~cm}$, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with $y$ component $B_{y}$. That component is to be measured as loop 2 is gradually moved in the positive direction of the $y$ axis. Figure 29-74b gives $B_{y}$ as a function of the position $y$ of loop 2. The curve approaches an asymptote of $B_{y}=7.20 \mu \mathrm{~T}$ as $y \rightarrow \infty$. The horizontal scale is set by $y_{s}=10.0 \mathrm{~cm}$. What are (a) current $i_{1}$ in loop 1 and (b) current $i_{2}$ in loop 2?


Figure 29-74 Problem 60.

- $\bullet 61$ A circular loop of radius 12 cm carries a current of 15 A . A flat coil of radius 0.82 cm , having 50 turns and a current of 1.3 A , is concentric with the loop. The plane of the loop is perpendicular to the plane of the coil. Assume the loop's magnetic field is uniform across the coil. What is the magnitude of (a) the magnetic field produced by the loop at its center and (b) the torque on the coil due to the loop?
- 62 In Fig. 29-75, current $i=56.2 \mathrm{~mA}$ is set up in a loop having two radial lengths and two semicircles of radii $a=5.72 \mathrm{~cm}$


Figure 29-75 Problem 62.
and $b=9.36 \mathrm{~cm}$ with a common center $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at $P$ and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?
-•63 In Fig. 29-76, a conductor carries 6.0 A along the closed path abcdefgha running along 8 of the 12 edges of a cube of edge length 10 cm . (a) Taking the path to be a combination of three square current loops ( $b c f g b, a b g h a$, and $c d e f c$ ), find the net magnetic moment of the path in unit-vector notation. (b) What is the magnitude of the net magnetic field at the $x y z$ coordinates of $(0,5.0 \mathrm{~m}, 0)$ ?


Figure 29-76 Problem 63.

## Additional Problems

64 In Fig. 29-77, a closed loop carries current $i=200 \mathrm{~mA}$. The loop consists of two radial straight wires and two concentric circular arcs of radii 2.00 m and 4.00 m . The angle $\theta$ is $\pi / 4$ rad. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the center of curvature $P$ ?
65 A cylindrical cable of radius


Figure 29-77 Problem 64. 8.00 mm carries a current of 25.0 A , uniformly spread over its cross-sectional area. At what distance from the center of the wire is there a point within the wire where the magnetic field magnitude is 0.100 mT ?

66 Two long wires lie in an $x y$ plane, and each carries a current in the positive direction of the $x$ axis. Wire 1 is at $y=10.0 \mathrm{~cm}$ and carries 6.00 A ; wire 2 is at $y=5.00 \mathrm{~cm}$ and carries 10.0 A . (a) In unit-vector notation, what is the net magnetic field $\vec{B}$ at the origin? (b) At what value of $y$ does $\vec{B}=0$ ? (c) If the current in wire 1 is reversed, at what value of $y$ does $\vec{B}=0$ ?
67 Two wires, both of length $L$, are formed into a circle and a square, and each carries current $i$. Show that the square produces a greater magnetic field at its center than the circle produces at its center.
68 A long straight wire carries a current of 50 A . An electron, traveling at $1.0 \times 10^{7} \mathrm{~m} / \mathrm{s}$, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?

69 Three long wires are parallel to a $z$ axis, and each carries a current of 10 A in the positive $z$ direction. Their points of intersection with the $x y$ plane form an equilateral triangle with sides of 50 cm , as shown in Fig. 29-78. A fourth wire (wire $b$ ) passes through the midpoint of the base of the triangle and is parallel to


Figure 29-78 Problem 69. the other three wires. If the net magnetic force on wire $a$ is zero, what are the (a) size and (b) direction $(+z$ or $-z$ ) of the current in wire $b$ ?

70 Figure 29-79 shows a closed loop with current $i=2.00 \mathrm{~A}$. The loop consists of a half-circle of radius 4.00 m , two quarter-circles each of radius 2.00 m , and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?

71 A 10-gauge bare copper wire


Figure 29-79 Problem 70. ( 2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

72 A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm . If the magnitude of the magnetic field at a point 5.0 mm from the wire is $1.0 \mu \mathrm{~T}$, what are the (a) size and (b) direction of the current in the wire?
73 Figure 29-80 shows a cross section of a long cylindrical conductor of radius $a=4.00 \mathrm{~cm}$ containing a long cylindrical hole of radius $b=1.50 \mathrm{~cm}$. The central axes of the cylinder and hole are parallel and are distance $d=2.00 \mathrm{~cm}$ apart; current $i=5.25 \mathrm{~A}$ is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases $b=0$ and $d=0$.


Figure 29-80
Problem 73.

74 The magnitude of the magnetic field at a point 88.0 cm from the central axis of a long straight wire is $7.30 \mu \mathrm{~T}$. What is the current in the wire?
75 SSM Figure 29-81 shows a wire segment of length $\Delta s=3.0 \mathrm{~cm}$, centered at the origin, carrying current $i=2.0 \mathrm{~A}$ in the positive $y$ direction (as part of some complete circuit). To calculate the magnitude of the magnetic field $\vec{B}$ produced by the segment at a point several meters from the origin, we can use $B=\left(\mu_{0} / 4 \pi\right) i \Delta s(\sin \theta) / r^{2}$ as the


Figure 29-81 Problem 75. Biot-Savart law. This is because $r$ and $\theta$ are essentially constant over the segment. Calculate $\vec{B}$ (in unit-vector notation) at the $(x, y, z)$ coordinates (a) $(0,0,5.0 \mathrm{~m})$, (b) $(0,6.0 \mathrm{~m}, 0)$, (c) ( $7.0 \mathrm{~m}, 7.0 \mathrm{~m}, 0$ ), and (d) ( $-3.0 \mathrm{~m},-4.0 \mathrm{~m}, 0$ ).

76 ©o Figure 29-82 shows, in cross section, two long parallel wires spaced by distance $d=10.0 \mathrm{~cm}$; each carries 100 A , out of the page in wire 1 . Point $P$ is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at $P$ if the current in wire 2 is (a) out of the page and (b) into the page?


Figure 29-82 Problem 76.

77 In Fig. 29-83, two infinitely long wires carry equal currents $i$. Each follows a $90^{\circ}$ arc on the circumference of the same circle of radius $R$. Show that the magnetic field $\vec{B}$ at the center of the circle is the same as the field $\vec{B}$ a distance $R$ below an infinite straight wire carrying a current $i$ to the left.

78 A long wire carrying 100 A is perpendicular to the magnetic field lines of


Figure 29-83
Problem 77. a uniform magnetic field of magnitude 5.0 mT . At what distance from the wire is the net magnetic field equal to zero?

79 A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm ) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm , (b) 3.0 mm , and (c) 5.0 mm from the central axis of the wire and cylinder?

80 A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?
81 SSM Figure 29-84 shows a cross section of an infinite conducting sheet carrying a current per unit $x$-length of $\lambda$; the current emerges perpendicularly out of the page. (a) Use the Biot-Savart law and


Figure 29-84 Problem 81. symmetry to show that for all points $P$ above the sheet and all points $P^{\prime}$ below it, the magnetic field $\vec{B}$ is parallel to the sheet and directed as shown. (b) Use Ampere's law to prove that $B=\frac{1}{2} \mu_{0} \lambda$ at all points $P$ and $P^{\prime}$.
82 Figure 29-85 shows, in cross section, two long parallel wires that are separated by distance $d=18.6 \mathrm{~cm}$. Each carries 4.23 A , out of the page in wire 1 and into the page in wire 2 . In unitvector notation, what is the net magnetic field at point $P$ at distance $R=34.2 \mathrm{~cm}$, due to the two currents?

83 SSM In unit-vector notation, what is the magnetic field at point $P$ in Fig. 29-86 if $i=10 \mathrm{~A}$ and $a=8.0 \mathrm{~cm}$ ? (Note that the wires are not long.)
84 Three long wires all lie in an $x y$ plane parallel to the $x$ axis. They are spaced equally, 10 cm apart. The two outer wires each carry a current of 5.0 A in the positive $x$ direction. What is the magnitude of the force on a 3.0 m section of either of the outer wires if the


Figure 29-85 Problem 82.


Figure 29-86 Problem 83.
current in the center wire is 3.2 A (a) in the positive $x$ direction and (b) in the negative $x$ direction?

85 SSM Figure 29-87 shows a cross section of a hollow cylindrical conductor of radii $a$ and $b$, carrying a uniformly distributed current $i$.
(a) Show that the magnetic field magnitude $B(r)$ for the radial distance $r$ in the range $b<r<a$ is given by

$$
B=\frac{\mu_{0} i}{2 \pi\left(a^{2}-b^{2}\right)} \frac{r^{2}-b^{2}}{r} .
$$



Figure 29-87
Problem 85.
(b) Show that when $r=a$, this equation gives the magnetic field magnitude $B$ at the surface of a long straight wire carrying current $i$; when $r=b$, it gives zero magnetic field; and when $b=0$, it gives the magnetic field inside a solid conductor of radius $a$ carrying current $i$. (c) Assume that $a=2.0 \mathrm{~cm}, b=1.8 \mathrm{~cm}$, and $i=100 \mathrm{~A}$, and then plot $B(r)$ for the range $0<r<6 \mathrm{~cm}$.

86 Show that the magnitude of the magnetic field produced at the center of a rectangular loop of wire of length $L$ and width $W$, carrying a current $i$, is

$$
B=\frac{2 \mu_{0} i}{\pi} \frac{\left(L^{2}+W^{2}\right)^{1 / 2}}{L W}
$$

87 Figure 29-88 shows a cross section of a long conducting coaxial cable and gives its radii $(a, b, c)$. Equal but opposite currents $i$ are uniformly distributed in the two conductors. Derive expressions for $B(r)$ with radial distance $r$ in the ranges (a) $r<c$, (b) $c<r<b$, (c) $b<r<a$, and (d) $r>a$. (e) Test these expressions for all the special cases that occur to you. (f) Assume that $a=2.0 \mathrm{~cm}, b=1.8 \mathrm{~cm}$,


Figure 29-88
Problem 87. $c=0.40 \mathrm{~cm}$, and $i=120 \mathrm{~A}$ and plot the function $B(r)$ over the range $0<r<3 \mathrm{~cm}$.
88 Figure 29-89 is an idealized schematic drawing of a rail gun. Projectile $P$ sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let $w$ be the distance between the rails, $R$ the radius of each rail, and $i$ the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$
F=\frac{i^{2} \mu_{0}}{2 \pi} \ln \frac{w+R}{R} .
$$

(b) If the projectile starts from the left end of the rails at rest, find the speed $v$ at which it is expelled at the right. Assume that $i=450 \mathrm{kA}, w=12 \mathrm{~mm}, R=6.7 \mathrm{~cm}, L=4.0 \mathrm{~m}$, and the projectile mass is 10 g .


Figure 29-89 Problem 88.

