Electrodynamics

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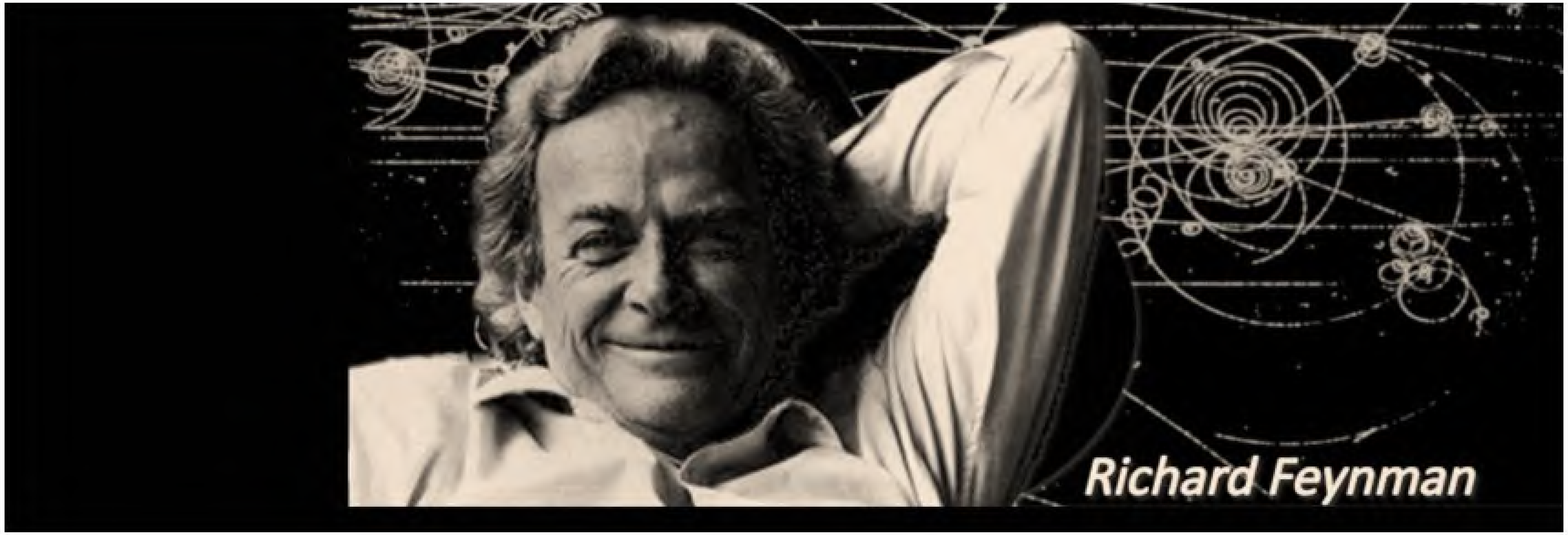
https://kanjouri.com



اگر همواره مانند گذشته بیندیشید، همیشه همان چیزهایی را به‌دست می‌آورید که تا کنون کسب کرده‌اید

فاینمن

درس سی‌اُم

بسط تابع گرین در مختصات استوانه‌ای

Expansion of Green Functions in Cylindrical Coordinates

دانشگاه خوارزمی

فرض کنید می‌خواهیم تابع گرین معادله‌ی پواسون را برای فضای تهی (بدون مرز) تعیین کنیم. در واقع معادل پتانسیل بار نقطه‌ای است.

$$\nabla^2 G(\mathbf{x}, \mathbf{x}') = -4\pi\delta(\mathbf{x} - \mathbf{x}')$$

$$\nabla^2 G(\rho, \phi, z; \rho', \phi', z') = -4\pi \frac{\delta(\rho - \rho')\delta(\phi - \phi')\delta(z - z')}{\rho}$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 G(\rho,\phi,z;\rho',\phi',z') = -4\pi\,\frac{\delta(\rho-\rho')\delta(\phi-\phi')\delta(z-z')}{\rho}$$

$$\delta(\phi-\phi') = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}$$

$$\delta(z-z') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik(z-z')}dk = \frac{1}{\pi}\int_{0}^{\infty}\cos k(z-z')dk$$

$$G(r,\phi,z;r',\phi',z') = \frac{1}{4\pi^2}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty} g_m(r,r')e^{ik(z-z')}e^{im(\phi-\phi')}dk$$

دانشگاه خوارزمی

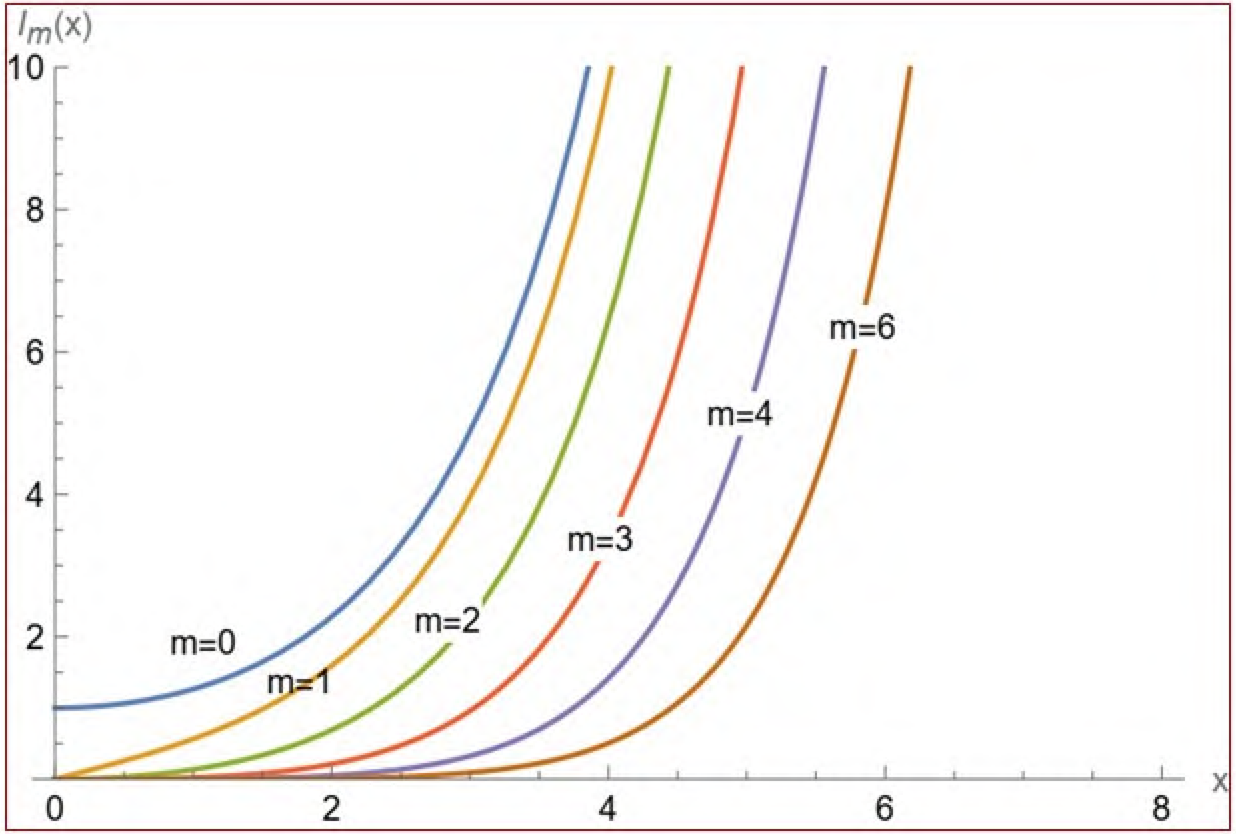
$$G(\rho,\phi,z;\rho',\phi',z') = \frac{1}{4\pi^2}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty} g_m(\rho,\rho')e^{ik(z-z')}e^{im(\phi-\phi')}dk$$

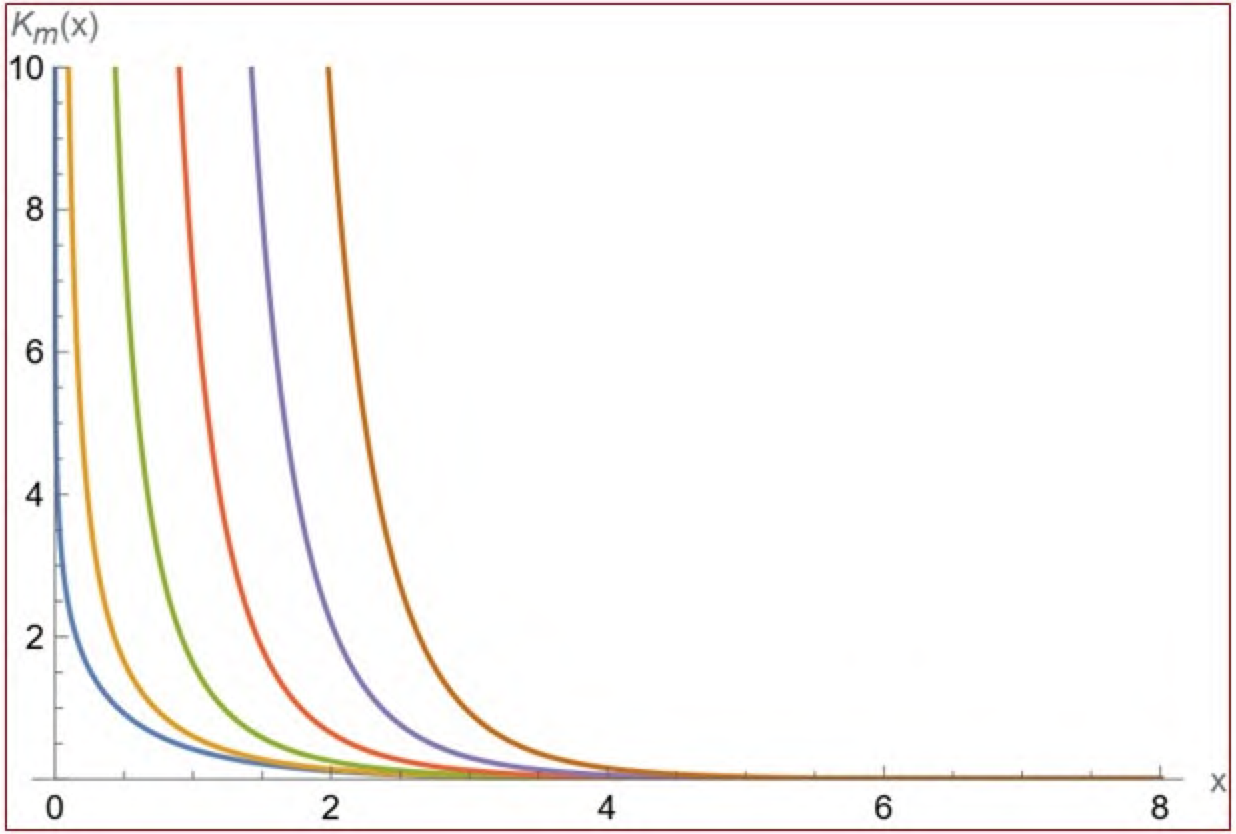
$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dg_m}{d\rho}\right) - \left(k^2 + \frac{m^2}{\rho^2}\right)g_m = -4\pi\frac{1}{\rho}\delta(\rho-\rho')$$

$\rho \neq \rho'$ $$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dg_m}{d\rho}\right) - \left(k^2 + \frac{m^2}{\rho^2}\right)g_m = 0$$

$$g_m = I_m(k\rho),\, K_m(k\rho)$$

$$g_m(\rho,\rho') = \begin{cases} A(\rho')I_m(k\rho) & \rho < \rho' \\ B(\rho')K_m(k\rho) & \rho > \rho' \end{cases}$$


$I_m(x)$
10
8
6
4
2
0
2
4
6
8
x
m=0
m=1
m=2
m=3
m=4
m=6


$K_m(x)$
10
8
6
4
2
0
2
4
6
8
x

$$g_m(\rho,\rho') = \begin{cases} A(\rho')I_m(k\rho) & \rho < \rho' \\ B(\rho')\,K_m(k\rho) & \rho > \rho' \end{cases}$$

$$g_m(\rho',\rho) = \begin{cases} A(\rho)I_m(k\rho') & \rho' < \rho \\ B(\rho)\,K_m(k\rho') & \rho' > \rho \end{cases}$$

$$g_m(\rho,\rho') = \begin{cases} CK_m(k\rho')I_m(k\rho) & \rho < \rho' \\ CI_m(k\rho')\,K_m(k\rho) & \rho > \rho' \end{cases}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dg_m}{d\rho}\right) - \left(k^2 + \frac{m^2}{\rho^2}\right)g_m = -4\pi\frac{1}{\rho}\delta(\rho-\rho')$$

$$\left(\frac{dg_m}{d\rho}\right)_{\rho'+0} - \left(\frac{dg_m}{d\rho}\right)_{\rho'-0} = -4\pi\frac{1}{\rho'}$$

دانشگاه خوارزمی

$$g_m(\rho,\rho') = \begin{cases} CK_m(k\rho')I_m(k\rho) & \rho < \rho' \\ \\ CI_m(k\rho')K_m(k\rho) & \rho > \rho' \end{cases}$$

$$\left(\frac{dg_m}{d\rho}\right)_{\rho'+0} - \left(\frac{dg_m}{d\rho}\right)_{\rho'-0} = -4\pi\,\frac{1}{\rho'}$$

$$C\Big[\,I_m(k\rho')\,K'_m(k\rho') - K_m(k\rho')I'_m(k\rho')\Big] = -4\pi\,\frac{1}{k\rho'}$$

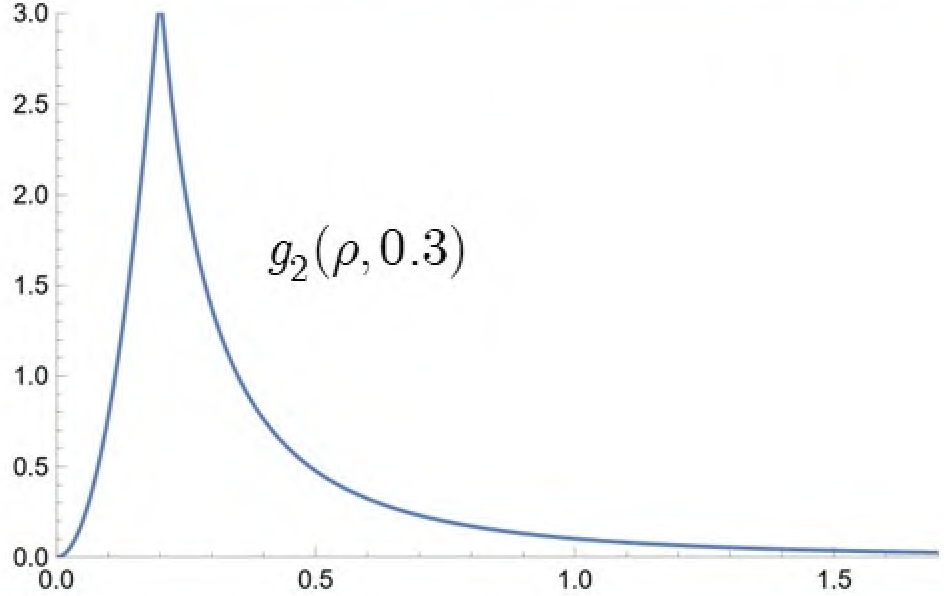
$$I_m(x)\,K'_m(x) - K_m(x)I'_m(x) = -\frac{1}{x}$$

$$C = 4\pi$$

دانشگاه خوارزمی

$$g_m(\rho,\rho') = \begin{cases} 4\pi K_m(k\rho')I_m(k\rho) & \rho < \rho' \\ \\ 4\pi I_m(k\rho')K_m(k\rho) & \rho > \rho' \end{cases}$$

$$g_m(\rho,\rho') = 4\pi K_m(k\rho_>)I_m(k\rho_<)$$



$$G(\rho,\phi,z;\rho',\phi',z') = \frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{\pi}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty} K_m(k\rho_>)I_m(k\rho_<)e^{ik(z-z')}e^{im(\phi-\phi')}dk$$

$$G(\rho,\phi,z;\rho',\phi',z') = \frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{2}{\pi}\sum_{m=-\infty}^{\infty}\int_{0}^{\infty} K_m(k\rho_>)I_m(k\rho_<)\cos k(z-z')e^{im(\phi-\phi')}dk$$

شاد و مهربان باشید